

Testing gravity with planetary ephemerides

Part II (ex part I): Gravity in weak-field

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It is also possible to write the metric tensor g such as g is a perturbation of the Minkowski metric

$$\boxed{g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)} \quad (108)$$

The Einstein equation can then be developed non-linearly and can take the following form :

$$\boxed{\square \bar{h}^{\mu\nu} - \partial^\mu \bar{H}^\nu - \partial^\nu \bar{H}^\mu + \eta^{\mu\nu} \partial_\rho \bar{H}^\rho = -\frac{16\pi G}{c^4} [T^{\mu\nu} + t^{\mu\nu}(\bar{h})] ,} \quad (109)$$

$$\square = \eta^{\rho\sigma} \partial_\rho \partial_\sigma = -c^{-2} \partial_t^2 + \Delta$$

d'Alembertian

$$\bar{H}^\mu = \partial_\rho \bar{h}^{\mu\rho}$$

With some appropriate variable transformations (called *gauge*), it is possible to obtain

$$\boxed{\square \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4} [T^{\mu\nu} + t^{\mu\nu}(\bar{h})] .} \quad (110a)$$

$$(110b) \quad t^{\mu\nu}(\bar{h}) = O(\bar{h}^2)$$

Definition: One calls the *weak field approximation* when $|h_{\mu\nu}| \approx \frac{2U}{c^2}$ and the velocities of the particles are small relative to c . (110)

More specifically, in the solar system where $|h_{\mu\nu}| \approx \frac{2U}{c^2} \leq 10^{-6}$ and $|v/c| \lesssim 10^{-4}$

In these conditions $\square = \eta^{\rho\sigma} \partial_\rho \partial_\sigma = -c^{-2} \partial_t^2 + \Delta$

$$\square \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4} [T^{\mu\nu} + t^{\mu\nu}(\bar{h})] .$$

$$T^{\mu\nu} = \rho u^\mu u^\nu .$$

$$\Delta U = -4\pi G \rho .$$

Eq. (110) can then be approximated with $\Delta \bar{h}^{\mu\nu} \simeq -\frac{16\pi G}{c^4} T^{\mu\nu}$ and the metric can be derived with Eq (94)

$$\Delta h^{\mu\nu} \simeq \frac{-4}{c^4} \Delta U u^\mu u^\nu \simeq \frac{-4}{c^4} \Delta U \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

where U is the newtonian potential and all terms in $(v/c)^4$ are neglected.

In the solar system, $U = \frac{GM_\odot}{r}$ where $M_\odot$ is the (gravitationnal) mass of the sun.

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$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

(111a)

$$\begin{aligned} g_{00} &= -1 + \frac{2U}{c^2} + \mathcal{O}(c^{-3}), \\ g_{0i} &= \mathcal{O}(c^{-2}), \\ g_{ij} &= \delta_{ij} + \mathcal{O}(c^{-1}), \end{aligned}$$

with $(v/c)^4$ neglected $\longrightarrow$

$$\begin{aligned} g_{00} &\simeq -1 + \frac{2U}{c^2}, \\ g_{0i} &\simeq 0, \\ g_{ij} &\simeq \delta_{ij} \left(1 + \frac{2U}{c^2} \right) \end{aligned}$$

(111b)

In the solar system, solving the equations of motion of the light is equivalent to study the photon free motion in the gravity field of the Sun. We will then look for solving the geodesic of photon in the Sun gravity field.

$$\boxed{\frac{d}{dp} \left(g_{\lambda\mu} \frac{dx^\mu}{dp} \right) = \frac{1}{2} \partial_\lambda g_{\mu\nu} \frac{dx^\mu}{dp} \frac{dx^\nu}{dp}} \quad (112)$$

$$\boxed{\frac{1}{2} g_{\mu\nu}(x) \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} = e ,}$$

In the case of the photon, we know that p cannot be the proper time because in this case $\tau = s = 0$. In any case, p is just an intermediate parameter that will disappear at the end.

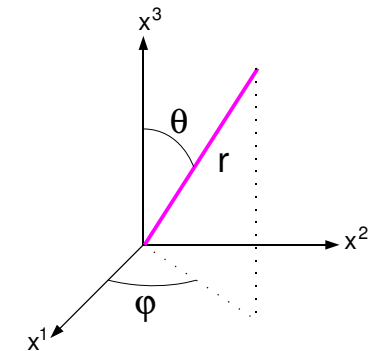
We consider to use the spherical coordinates $\{t, r, \theta, \varphi, \psi\}$ instead of $\{t, x_1, x_2, x_3\}$

$$\boxed{\begin{aligned} g_{00} &\simeq -1 + \frac{2U}{c^2} , \\ g_{0i} &\simeq 0 , \\ g_{ij} &\simeq \delta_{ij} \left(1 + \frac{2U}{c^2} \right) \end{aligned}}$$



$$\begin{aligned} g_{00} &\simeq -1 + \frac{2U}{c^2} \\ g_{rr} &\simeq 1 + \frac{2U}{c^2} , \\ g_{\theta\theta} &\simeq \left(1 + \frac{2U}{c^2} \right) r^2 , \\ g_{\varphi\varphi} &\simeq \left(1 + \frac{2U}{c^2} \right) r^2 \sin^2 \theta \end{aligned}$$

(113)



g_{ij} does not depend on φ and t .

We write Eq. (112) for each of the coordinates $\{t, r, \theta, \varphi, \psi\}$ but we note that the metric does not depend on φ and t .

→ φ and t are ignorable meaning that in the case of λ being ignorable $g_{\lambda\mu}dx^\mu/dp = \text{const}$

- Equation in t : $\left(-1 + \frac{2U}{c^2}\right) \frac{dt}{dp} = \text{const}$ (114a)

- Equation in φ : $\left(1 + \frac{2U}{c^2}\right) r^2 \sin^2 \theta \frac{d\varphi}{dp} = \text{const}$ (114b)

- Equation in θ : $\frac{d}{dp} \left\{ \left(1 + \frac{2U}{c^2}\right) r^2 \frac{d\theta}{dp} \right\} = \left(1 + \frac{2U}{c^2}\right) r^2 \sin \theta \cos \theta \left(\frac{d\varphi}{dp}\right)^2$ (114c)

Eq. (114c) has an obvious solution for $\theta = \frac{\pi}{2}$

- Equation in r : We choose for r to use the equation of energy with $e=0$ following Eq. (49)

$$\boxed{\frac{1}{2} g_{\mu\nu}(x) \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} = e,}$$

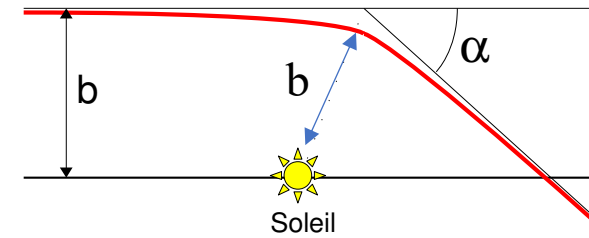
With $ds^2=0$, it comes $\left(1 + \frac{2U}{c^2}\right) \left[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)\right] = \left(1 - \frac{2U}{c^2}\right) c^2 dt^2$ (114d)

- (114b) / (114a) gives in neglecting the terms in $(U/c^2)^2$

Equation of conservation
of angular momentum

$$r^2 \frac{d\varphi}{dt} = b \left(1 + \frac{4U}{c^2} \right) \quad (115)$$

Impact parameter, the minimal distance between the sun and the photon

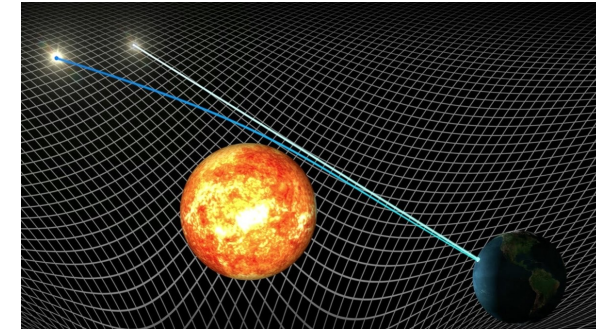


- In re-injecting Eq. (115) in (114d), one obtains the equation of the trajectory $r(\varphi)$

$$\left[\frac{d}{d\varphi} \left(\frac{b}{r} \right) \right]^2 + \left(\frac{b}{r} \right)^2 = 1 + \frac{4U}{c^2} \quad (116)$$

Eq. (116) is the equation of an hyperbole which as a solution with φ_0 is the original direction and α is the deviation angle

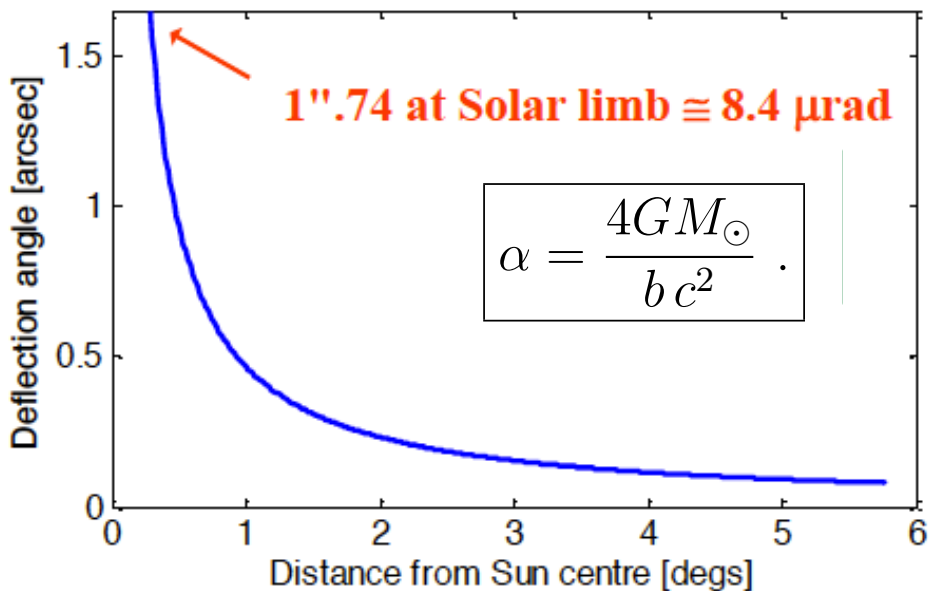
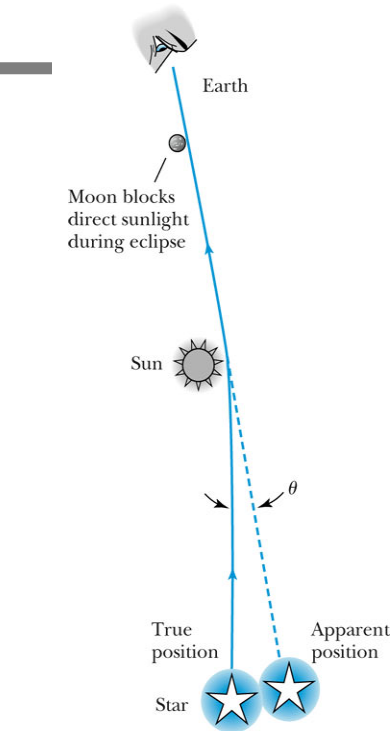
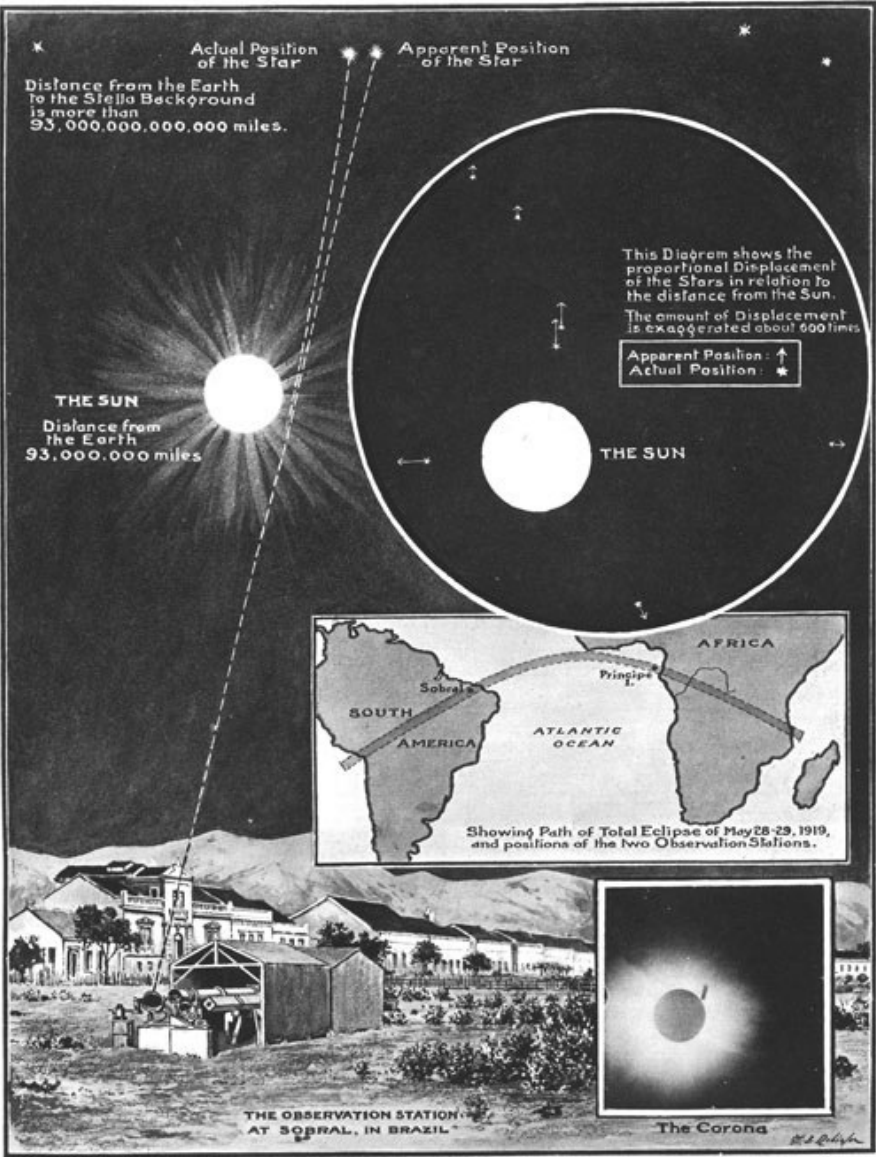
The deviation angle is max when b is min, so b =sun radius.



$$\frac{b}{r} = \sin (\varphi - \varphi_0) + \frac{\alpha}{2} \quad (117a)$$

$$\alpha = \frac{4GM_{\odot}}{b c^2} . \quad (117b)$$

$$\alpha_{\max} = \frac{4GM_{\odot}}{R_{\odot}c^2} = 1.75''$$



SCLERA: an Astrometric Telescope for Experimental Relativity

APPLIED OPTICS / Vol. 13, No. 1 / January 1974

J. R. Oleson, C. A. Zanoni, H. A. Hill, A. W. Healy, P. D. Clayton, and D. L. Patz



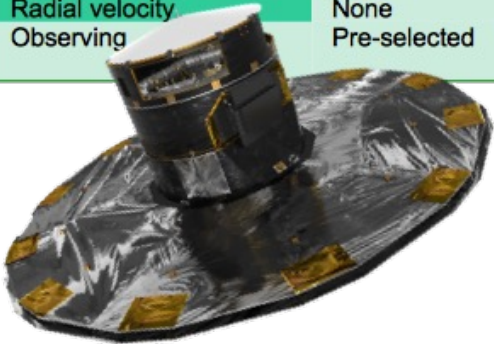
An f/100, 12.2-m focal length photoelectric telescope designed specifically for daytime astrometry of objects near the sun is now operative at its Tucson, Arizona, site. The design goal was to achieve accuracies of order 0.001 sec of arc in field position measurements of stars. To accomplish this, many features reducing systematic and random errors are employed, including Schupmann medial telescope optics, compensation for lateral color aberration, apodization for reduction of diffracted light, and use of an accurately measured solar diameter for calibrating the field.

Precision limitations from ground at the few milli-arcsec level...

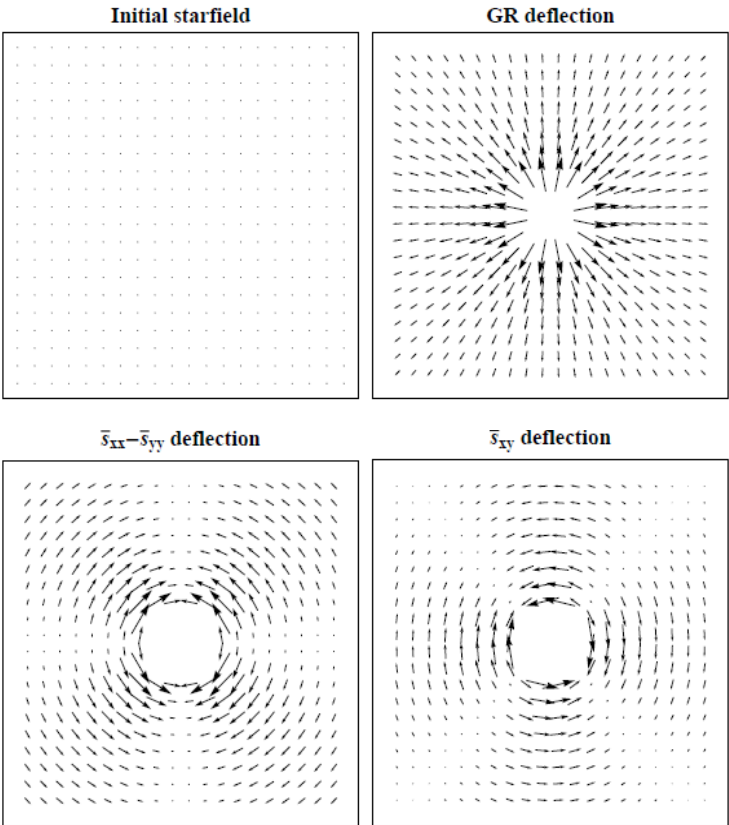
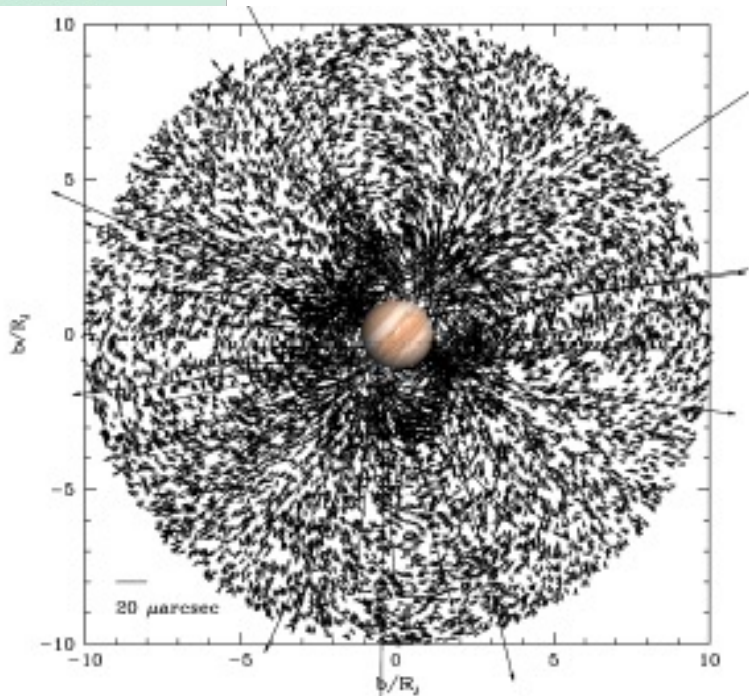
Authors	Year	Deflection ["]
Dyson & al.	1920	1.98 ± 0.16
Dodwell & al.	1922	1.77 ± 0.40
Freundlich & al.	1929	2.24 ± 0.10
Mikhailov	1936	2.73 ± 0.31
van Biesbroeck	1947	2.01 ± 0.27
van Biesbroeck	1952	1.70 ± 0.10
Schmeidler	1959	2.17 ± 0.34
Schmeidler	1961	1.98 ± 0.46
TMET	1973	1.66 ± 0.19

Chapter 1: Weak field approximation

	Hipparcos	Gaia
Magnitude limit	12 mag	20 mag
Completeness	7.3 – 9.0 mag	20 mag
Bright limit	0 mag	3 mag (assessment for brighter stars ongoing)
Number of objects	120,000	47 million to G = 15 mag 360 million to G = 18 mag 1192 million to G = 20 mag
Effective distance limit	1 kpc	50 kpc
Quasars	1 (3C 273)	500,000
Asteroids	48 main belt	250,000 main belt + 50 KBO
Galaxies	None	1,000,000
Accuracy	1 milliarcsec	7 μ arcsec at G = 10 mag 26 μ arcsec at G = 15 mag 600 μ arcsec at G = 20 mag
Photometry	2-colour (B and V)	Low-res. spectra to G = 20 mag
Radial velocity	None	15 km s ⁻¹ to GRVS = 16 mag
Observing	Pre-selected	Complete and unbiased

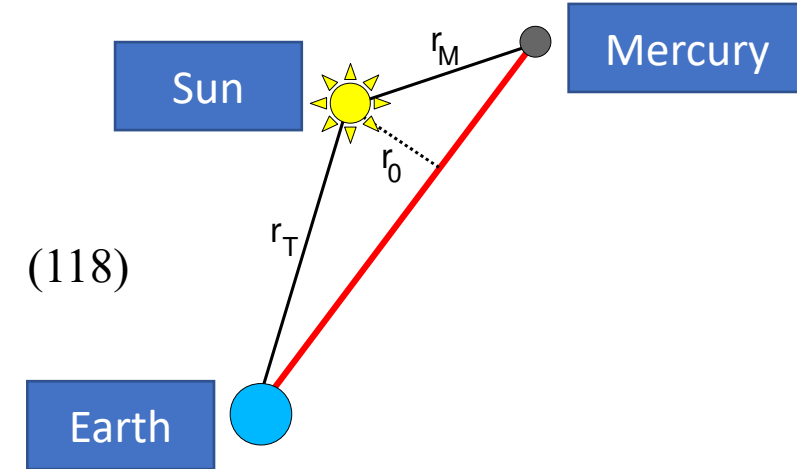


ight



- By recombining (115) and (114d), we can ride off φ and in neglecting U/c^2 terms, we have

$$\left(\frac{dr}{cdt}\right)^2 = \left(1 - \frac{r_0^2}{r^2}\right) \left\{1 - \frac{4U}{c^2} \left(1 + \frac{r_0}{r + r_0}\right)\right\} \quad (118)$$

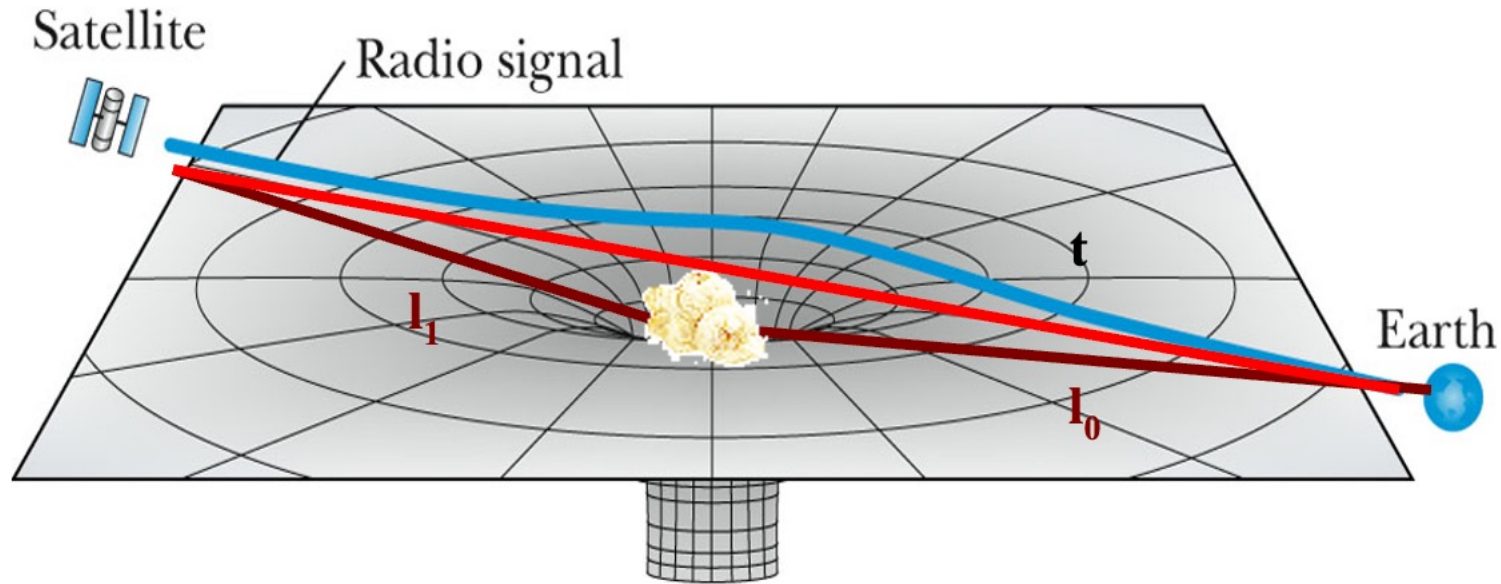


- In integrating Eq. (118) knowing r_T and r_M we can compute T , the time delay between the transmission of a signal from earth and its reception at Mercury with

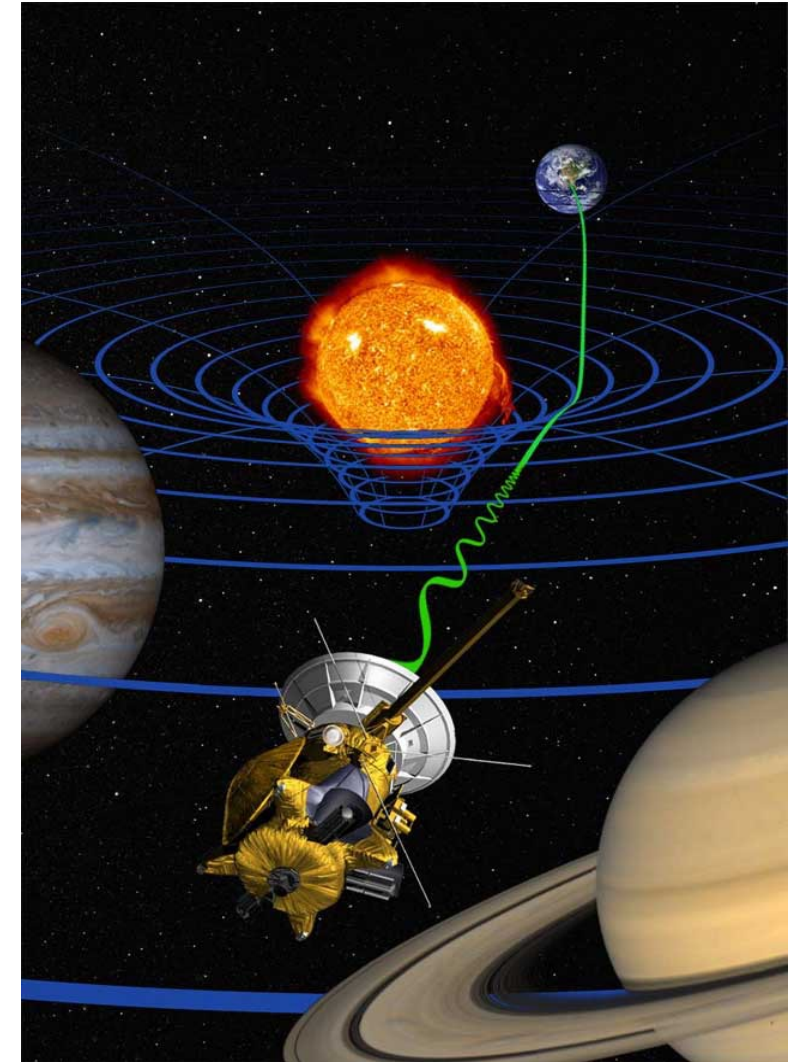
$$T = \frac{2}{c} \left\{ \underbrace{\sqrt{r_T^2 - r_0^2} + \sqrt{r_M^2 - r_0^2}}_{\text{Time delay w/o general relativity}} + \underbrace{\frac{4GM_\odot}{c^2} \left(\sqrt{\frac{r_T - r_0}{r_T + r_0}} + \sqrt{\frac{r_M - r_0}{r_M + r_0}} \right) + \frac{4GM_\odot}{c^2} \ln \left[\frac{(r_T + \sqrt{r_T^2 - r_0^2})(r_M + \sqrt{r_M^2 - r_0^2})}{r_0^2} \right]}_{\text{Time delay with general relativity}} \right\} \quad (119)$$

Time delay w/o general relativity, like if the photon path is a direct line

Time delay with general relativity with the curvature of space-time



For the Earth-Sun-Mercury configuration the max effect is 259 microseconds of time.



In the solar system, solving the equations of motion of the planets is equivalent to study their free motion in the gravity field of the Sun. We will then look for solving the geodesic of point-mass particule in the Sun gravity field.

$$\frac{d}{dp} \left(g_{\lambda\mu} \frac{dx^\mu}{dp} \right) = \frac{1}{2} \partial_\lambda g_{\mu\nu} \frac{dx^\mu}{dp} \frac{dx^\nu}{dp}$$

$$T^{\mu\nu} = \sum_A \frac{m_A u^0/c}{\sqrt{-g}} v_A^\mu v_A^\nu \delta^{(3)}(\mathbf{x} - \mathbf{x}_A(t)),$$

In the case of planets, p can be substitute to proper time.

For the planets, we will use the post-newtonian harmonic gauge metric

(121)

$$\begin{aligned} g_{00} &= -1 + \frac{2U}{c^2} + \mathcal{O}(c^{-3}), \\ g_{0i} &= \mathcal{O}(c^{-2}), \\ g_{ij} &= \delta_{ij} + \mathcal{O}(c^{-1}), \end{aligned}$$

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}), \\ g_{0i} &= -\frac{4w^i}{c^3} \mathcal{O}(c^{-4}), \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2w}{c^2} \right) + \mathcal{O}(c^{-3}). \end{aligned}$$

with $\left\{ \begin{aligned} w_0 &= \sum_A \frac{Gm_A}{r_A}, \\ w &= w_0 - \frac{1}{c^2} \Delta + \mathcal{O}(c^{-4}), \\ w^i &= \sum_A \frac{Gm_A}{r_A} v_A^i + \mathcal{O}(c^{-2}), \end{aligned} \right.$

$$\Delta = \sum_A \frac{Gm_A}{r_A} \left\{ -2v_a^2 + \sum_{B \neq A} \frac{Gm_B}{r_{BA}} + \frac{1}{2} \left[\frac{(r_A^k v_A^k)^2}{r_A^2} + r_A^k a_A^k \right] \right\}$$

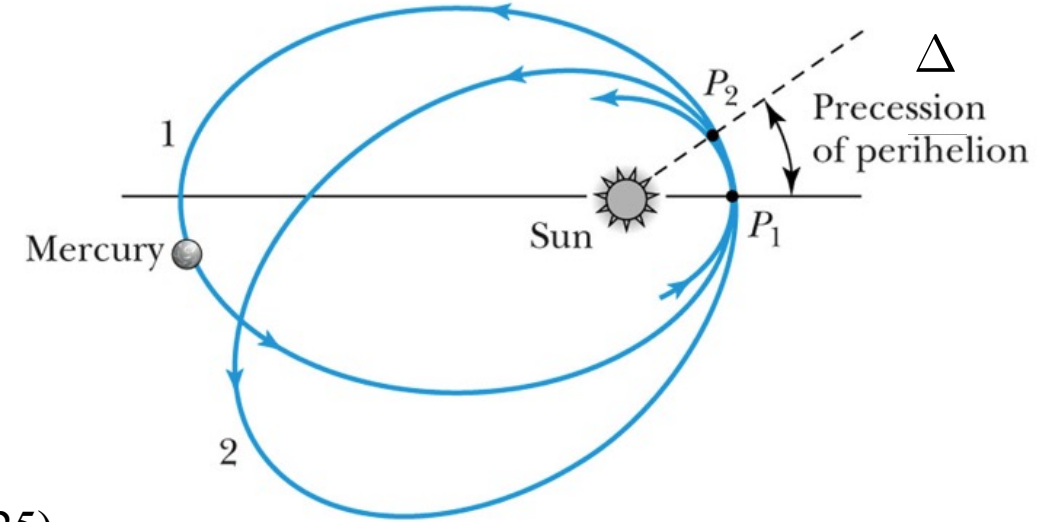
With spherical coordinates, only the g_{00} term of the metric is changed as in eq. (121). As we did with the deflection of light, we can consider the equations for each of the coordinates, with φ and t ignorable.

We then obtain the equation of the trajectory of the planets orbiting the sun in spherical coordinates

$$\left(\frac{d}{d\varphi} \left(\frac{1}{r} \right) \right)^2 = \left(1 - \frac{\Delta}{\pi} \right) \left(\frac{1}{r} - \frac{1}{r_A} \right) \left(\frac{1}{r_P} - \frac{1}{r} \right) \quad (125)$$

$$\Delta = \frac{6\pi G M_{\odot}}{c^2 a (1 - e^2)} \quad (126)$$

Distances at aphelia $r_A = a(1+e)$ and perihelia $r_P = a(1-e)$



At first order of approximation of U , one has
 $\rightarrow$ It is the eq. of a ellipse, precessing at each orbit by Δ

$$r = \frac{a(1 - e^2)}{1 + e \cos \left[\left(1 - \frac{\Delta}{2\pi} \right) (\varphi - \varphi_0) \right]} \quad (127)$$

$$r = \frac{a(1 - e^2)}{1 + e \cos \left[\left(1 - \frac{\Delta}{2\pi}\right) (\varphi - \varphi_0) \right]}$$

$$\Delta = \frac{6\pi G M_{\odot}}{c^2 a (1 - e^2)} \cdot$$

For Mercury, $a=58 \times 10^6$ km,
 $e=0.2$ with a period of 0.24 years
 $\rightarrow \Delta = 43''$ per century

Ephemerides	Gaillot Gaillot and Le Verrier (1913)		DE102 Standish (1983)		DE440/INPOP19a Park et al. (2021) Fienga et al. (2019a)		GR
Data span	1800-1913		1913-1983		1924-2021		
	angle	distance Earth-	angle	distance Earth-	angle	distance Earth-	$\Delta \dot{\omega}$
	''	km	''	km	''	km	'' .yr ⁻¹
Mercury	1	450	0.050	5	0.002	0.004	0.43
Venus	0.5	100	0.050	2	0.002	0.006	0.14
Mars	0.5	150	0.050	0.050	0.001	0.0015	0.065
Jupiter	0.5	1400	0.1	10	0.010	0.020	0.019
Saturn	0.5	3000	0.1	600	0.001	0.020	0.010
Uranus	1	12700	0.2	2540	0.050	10	0.005
Neptune	1	22000	0.2	4400	0.050	50	0.0033
Pluto	1	24000	0.2	4800	0.050	2400	0.0027

$$L = mc^2 \sqrt{-g_{00} - v^i g_{0i} - v^i v^j g_{ij}}$$

The method is then to use the Lagrangian to derive the equation of motion. For this, we will use

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}), \\ g_{0i} &= -\frac{4w^i}{c^3} \mathcal{O}(c^{-4}), \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2w}{c^2} \right) + \mathcal{O}(c^{-3}). \end{aligned} \quad (122)$$

Now, injecting the metric g of Eq. (121) into the definition of the Lagrangian of motion one gets for a test particle B the following Lagrangian of motion

$$\begin{aligned} L_B &= -m_B c^2 + m_B \frac{v_B^2}{2} + \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} + \frac{m_B v_B^4}{8c^2} \\ &\quad + \frac{1}{c^2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[-4 \mathbf{v}_A \cdot \mathbf{v}_B + \frac{3}{2} v_B^2 + 2v_A^2 - \frac{(\mathbf{v}_A \cdot \mathbf{r}_{AB})^2}{2r_{AB}^2} - \frac{\mathbf{r}_{AB} \cdot \mathbf{a}_A}{2} \right] \\ &\quad - \frac{1}{c^2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[\sum_{D \neq A} \frac{G m_D}{r_{AD}} + \sum_{D \neq B} \frac{1}{2} \frac{G m_D}{r_{DB}} \right] + \mathcal{O}(c^{-4}). \end{aligned}$$

with Euler-Lagrange equations

$$\frac{d}{dt} \left(\frac{\partial L_B}{\partial v} \right) - \frac{\partial L_B}{\partial x} = 0$$

We obtain the equation of motion explored first by Lorentz and Droste and re-derived later by Einstein, Infeld and Hoffman

$$\begin{aligned}
 \mathbf{a}_T = & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3} \mathbf{r}_{AT} \\
 & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3 c^2} \mathbf{r}_{AT} \left\{ v_T^2 + 2v_A^2 - 4\mathbf{v}_A \cdot \mathbf{v}_T - \frac{3}{2} \left(\frac{\mathbf{r}_{AT} \cdot \mathbf{v}_A}{r_{AT}} \right)^2 \right. \\
 & \quad \left. - \frac{1}{2} \mathbf{r}_{AT} \cdot \mathbf{a}_A - 4 \sum_{B \neq T} \frac{\mu_B}{r_{TB}} - \sum_{B \neq A} \frac{\mu_B}{r_{AB}} \right\} \\
 & + \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} [4\mathbf{r}_{AT} \cdot \mathbf{v}_T - 3\mathbf{r}_{AT} \cdot \mathbf{v}_A] (\mathbf{v}_T - \mathbf{v}_A) \\
 & + \frac{7}{2} \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}} \mathbf{a}_A,
 \end{aligned} \tag{124}$$

Some examples of alternative theories

In this lecture we will consider two types of theories : the *metric theories* and the *non-metric theories*.

Let first consider the *metric theories* :

- Additional gravitational fields do not couple to matter directly $\rightarrow$ gravity is fully described by the metric,
- By construction, they respect the WEP
- All the differences with respect to GRT can be taken into account through constant parameters that appear in front of the gravitational potentials in the metric
- particles still follow geodesics of the metric and proper time is still defined from the metric only

In consequence, their differences with respect to general relativity can be parametrized by a set of a few constant parameters in the metric only $\rightarrow$ Parametrized Post-Newtonian (PPN) formalism

Parametrized Post-Newtonian (PPN) formalism → Small departure from GRT parametrized by 2 parameters: β and γ

GRT

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}), \\ g_{0i} &= -\frac{4w^i}{c^3} \mathcal{O}(c^{-4}), \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2w}{c^2} \right) + \mathcal{O}(c^{-3}). \end{aligned}$$

$$\beta = \gamma = 1$$



PPN

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2\beta w^2}{c^4} + \mathcal{O}(c^{-5}) \\ g_{0i} &= -\frac{(4\gamma + 3)}{2} \frac{w_i}{c^3} + \mathcal{O}(c^{-4}) \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2\gamma w}{c^2} \right) + \mathcal{O}(c^{-3}), \end{aligned}$$

(134)

Extended Parametrized Post-Newtonian (EPPN) formalism → Up to 6 additional parameters, not constrained in the SS so far.

Parametrized Post-Newtonian Framework

In GRT, $\beta = \gamma = 1$

$$\begin{aligned}
 \mathbf{a}_T = & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3} \mathbf{r}_{AT} \\
 & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3 c^2} \mathbf{r}_{AT} \left\{ \gamma v_T^2 + (\gamma + 1) v_A^2 - 2(1 + \gamma) \mathbf{v}_A \cdot \mathbf{v}_T - \frac{3}{2} \left(\frac{\mathbf{r}_{AT} \cdot \mathbf{v}_A}{r_{AT}} \right)^2 \right. \\
 & \quad \left. - \frac{1}{2} \mathbf{r}_{AT} \cdot \mathbf{a}_A - 2(\gamma + \beta) \sum_{B \neq T} \frac{\mu_B}{r_{TB}} - (2\beta - 1) \sum_{B \neq A} \frac{\mu_B}{r_{AB}} \right\} \\
 & + \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} [2(1 + \gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_T - (1 + 2\gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_A] (\mathbf{v}_T - \mathbf{v}_A) \\
 & + \frac{3 + 4\gamma}{2} \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}} \mathbf{a}_A,
 \end{aligned} \tag{16}$$

(Bertotti et al. 2003)

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$c(t_r - t_e)_{GRT} = R + \sum_A (\gamma + 1) \frac{\mu_A}{c^2} \ln \frac{\mathbf{n} \cdot \mathbf{r}_{rA} + r_{rA} + 2 \frac{(1 + \gamma) \mu_A}{c^2}}{\mathbf{n} \cdot \mathbf{r}_{eA} + r_{eA} + 2 \frac{(1 + \gamma) \mu_A}{c^2}}$$

$$\mathbf{a}_{LT} = (1 + \gamma) \frac{GS}{c^2 r^3} \left[\frac{3 \mathbf{k} \cdot \mathbf{r}}{r^2} (\mathbf{r} \wedge \mathbf{v}) - (\mathbf{k} \wedge \mathbf{v}) \right]$$



PPN planetary modeling

$$\mathbf{a}_{J_2^\odot} = -\nabla \left(\frac{\mu_\odot}{r^3} \frac{J_2^\odot R_\odot^2}{2} (3 \sin^2 \psi - 1) \right)$$

$$\mathbf{a}_T^{\text{total}} = \mathbf{a}_T(\mu_{SS}, \gamma, \beta, IC_{SS}) + \mathbf{a}_{J_2^\odot}(\mu_\odot, J_2^\odot) + \mathbf{a}_{LT}(S_\odot, \gamma),$$

$$\Delta \dot{\omega}_{PLA} = \mu_\odot(t) \times \left(\frac{2}{3} \frac{(2\gamma - \beta + 2)}{a(1 - e^2)c^2} + \Delta \dot{\omega}_{LT}(S, \gamma) \right) + J_2^\odot \frac{R_\odot^2}{a^2(1 - e^2)^2} + \Delta \dot{\omega}_{AST}$$

$$\Delta \dot{\omega}_{Moon} = \frac{(2\gamma - \beta + 2)GM_\odot(t)}{a(1 - e^2)c^2} + \Delta \dot{\omega}_{GEO} + \Delta \dot{\omega}_{SEL} + \Delta \dot{\omega}_{S,PLA}$$

$$\vec{a}_{LT}^\odot = \frac{(\gamma + 1)GS}{c^2 r^3} \left[\frac{3\vec{k} \cdot \vec{r}}{r^2} (\vec{r} \wedge \vec{v}) - (\vec{k} \wedge \vec{v}) \right] \quad (\text{INPOP20a+})$$

$$\Delta t_{SHAP} = (1 + \gamma) \frac{GM_\odot(t)}{c^3} \ln \left[\frac{l_0 + l_1 + t + \frac{(1 + \gamma)GM_\odot(t)}{c^2}}{l_0 + l_1 - t + \frac{(1 + \gamma)GM_\odot(t)}{c^2}} \right]$$

PPN planetary modeling

$$\mathbf{a}_T^{\text{total}} = \mathbf{a}_T(\mu_{SS}, \gamma, \beta, IC_{SS}) + \mathbf{a}_{J_2^\odot}(\mu_\odot, J_2^\odot) + \mathbf{a}_{LT}(S_\odot, \gamma),$$

$$\left(\frac{m^G}{m^I}\right)_T^{SEP} = 1 + \eta \times \frac{3}{5} \frac{Gm_T^G}{c^2 R_T},$$

$$\eta = 4(\beta - 1) - (\gamma - 1)$$

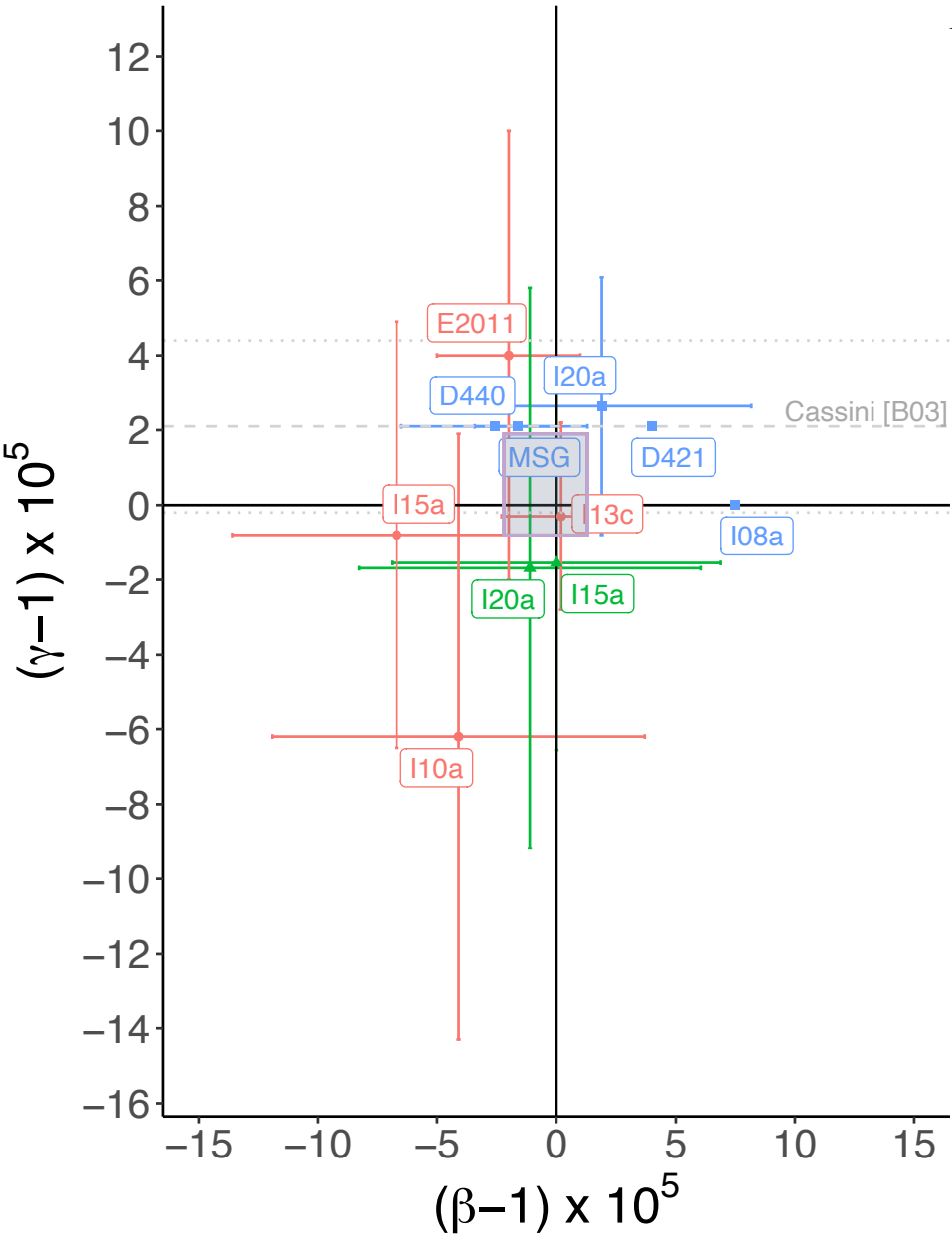
used so far only for
simulation or MSG
(Genova et al. 2018)

$$\Delta \dot{\omega}_{PLA} = \mu_\odot(t) \times \left(\frac{2}{3} \frac{(2\gamma - \beta + 2)}{a(1-e^2)c^2} + \Delta \dot{\omega}_{LT}(S, \gamma) \right) + J_2^\odot \frac{R_\odot^2}{a^2(1-e^2)^2} + \Delta \dot{\omega}_{AST}$$

$$\Delta \dot{\omega}_{Moon} = \frac{(2\gamma - \beta + 2)GM_\odot(t)}{a(1-e^2)c^2} + \Delta \dot{\omega}_{GEO} + \Delta \dot{\omega}_{SEL} + \Delta \dot{\omega}_{S,PLA}$$

$$\vec{a}_{LT}^\odot = \frac{(\gamma + 1)GS}{c^2 r^3} \left[\frac{3\vec{k} \cdot \vec{r}}{r^2} (\vec{r} \wedge \vec{v}) - (\vec{k} \wedge \vec{v}) \right] \quad (\text{INPOP20a+})$$

$$\Delta t_{SHAP} = (1 + \gamma) \frac{GM_\odot(t)}{c^3} \ln \left[\frac{l_0 + l_1 + t + \frac{(1+\gamma)GM_\odot(t)}{c^2}}{l_0 + l_1 - t + \frac{(1+\gamma)GM_\odot(t)}{c^2}} \right]$$



Frequentist approach

Full Fit or Partial Fit =
estimation by least
squares with all
parameters or just few
are free

Methods

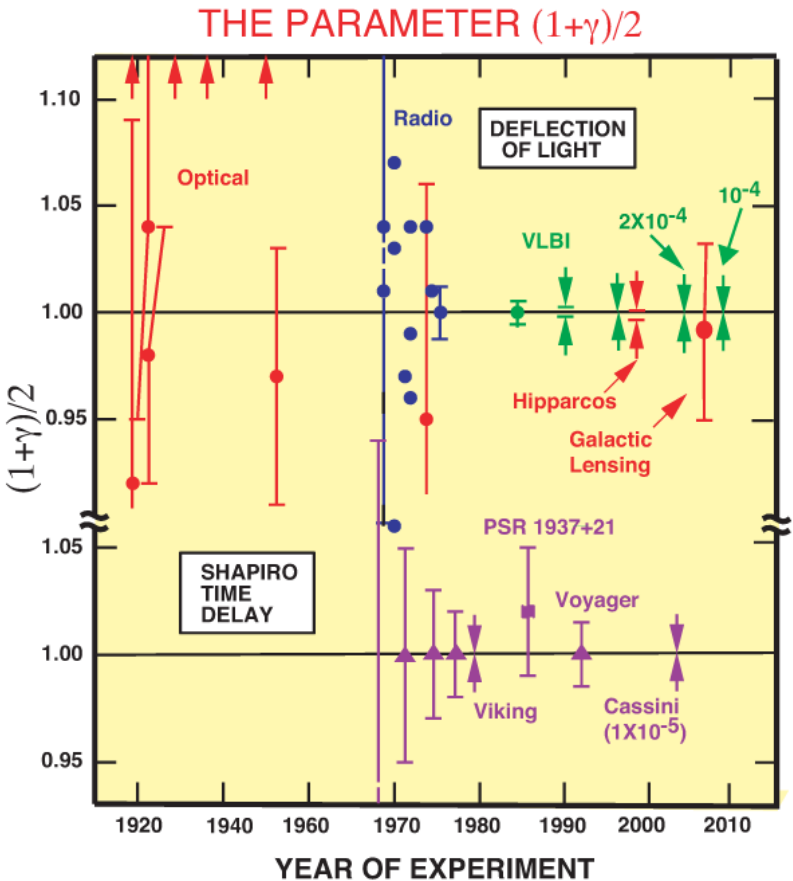
- Full Fit
- RW
- Partial Fit

Bayesian approach

RW = Random Walk=
Random exploration of
values for (β, γ) . The
values are selected
according to a « cost-
function »

$$(\beta - 1) = (-0.45 \pm 1.75) \times 10^{-5}$$

$$(\gamma - 1) = (0.55 \pm 1.35) \times 10^{-5}$$



Equivalence principal / Universality of Free Fall

Generic expression for both metric and non-metric theories

$$\begin{aligned}\Delta \mathbf{a}^{\overline{\text{UFF}}} &\equiv (\mathbf{a}_M - \mathbf{a}_E)^{\overline{\text{UFF}}} \\ &\approx Gm_S^G \left[\frac{\mathbf{r}_{SE}}{r_{SE}^3} \left(\left(\frac{m^G}{m^I} \right)_E - 1 \right) - \frac{\mathbf{r}_{SM}}{r_{SM}^3} \left(\left(\frac{m^G}{m^I} \right)_M - 1 \right) \right] \\ &\approx \mathbf{a}_E \left[\left(\left(\frac{m^G}{m^I} \right)_E - 1 \right) - \left(\left(\frac{m^G}{m^I} \right)_M - 1 \right) \right] \\ &\equiv \mathbf{a}_E \Delta_{\text{ESM}}\end{aligned}$$

with

$$\Delta_{\text{ESM}} = \left[\left(\frac{m^G}{m^I} \right)_E - \left(\frac{m^G}{m^I} \right)_M \right]. \quad (9)$$

Estimated with LLR

$$\mathbf{a}_T = - \left(\frac{m^G}{m^I} \right)_T \sum_{A \neq T} \frac{Gm_A^G}{r_{AT}^3} \mathbf{r}_{AT},$$

$$\Delta_{\text{ESM}} = \Delta_{\text{ESM}}^{\text{WEP}} + \Delta_{\text{ESM}}^{\text{SEP}}$$

Estimated with specific
experiments such as
Torsion Balance,
MICROSCOPE

Equivalence principal / Universality of Free Fall

$$\begin{aligned}\Delta \mathbf{a}^{\overline{\text{UFF}}} &\equiv (\mathbf{a}_M - \mathbf{a}_E)^{\overline{\text{UFF}}} \\ &\approx Gm_S^G \left[\frac{\mathbf{r}_{SE}}{r_{SE}^3} \left(\left(\frac{m^G}{m^I} \right)_E - 1 \right) - \frac{\mathbf{r}_{SM}}{r_{SM}^3} \left(\left(\frac{m^G}{m^I} \right)_M - 1 \right) \right] \\ &\approx \mathbf{a}_E \left[\left(\left(\frac{m^G}{m^I} \right)_E - 1 \right) - \left(\left(\frac{m^G}{m^I} \right)_M - 1 \right) \right] \\ &\equiv \mathbf{a}_E \Delta_{\text{ESM}}\end{aligned}$$

with

$$\Delta_{\text{ESM}} = \left[\left(\frac{m^G}{m^I} \right)_E - \left(\frac{m^G}{m^I} \right)_M \right].$$

Estimated with LLR

- Metric theories : WEP valid by definition ; only SEP

$$\left(\frac{m^G}{m^I} \right)_T^{SEP} = 1 + \eta \times \frac{3}{5} \frac{Gm_T^G}{c^2 R_T},$$

$$\eta = 4(\beta - 1) - (\gamma - 1)$$

$$\Delta_{\text{ESM}}^{\text{SEP}} = \eta_{\text{SEP}} \left[\left(\frac{|\Omega|}{mc^2} \right)_E - \left(\frac{|\Omega|}{mc^2} \right)_M \right]$$

Estimated with LLR

Self binding energy

Equivalence principal with LLR

Experiment	Type of test		
Torsion balance	WEP	$\Delta a < 10^{-13}$	Adelberger et al. 2003
MICROSCOPE	WEP	$\Delta a < 10^{-15}$	Touboul et al. 2019
LLR	GWEP, UFF, Metric theories	$\Delta a < 10^{-14}$ $\eta < 5 \times 10^{-5}$	Biskupek et al. 2021, Viswanathan et al. 2018
Messenger	SEP, metric theories	$\eta < 7 \times 10^{-5}$	Genova et al. 2021
PLE	SEP in Brans-Dicke framework	$ \eta \leq 3 \times 10^{-5}$	Mariani et al. 2024

Overview

- 1) General introduction to the planetary and lunar ephemerides (PLE)
 - Basics
 - GR framework
 - Accuracies
- 2) Tests in the PPN framework
 - Framework
 - Results and issues
 - Equivalence Principal
- 3) Other alternative theories
 - Time varying constants
 - Massive graviton, Yukawa suppression
 - Einstein Dilaton
 - Dark matter
- 4) Conclusions

Variation of G

- What is estimated is $\mu_{\odot} = GM_{\odot}$

$$\frac{\dot{\mu}_{\odot}}{\mu_{\odot}} = \frac{\dot{M}_{\odot}}{M_{\odot}} + \frac{\dot{G}}{G}$$

- Integrated equations are at each step

$$M_{\odot}(t) = M_{\odot}(J2000) + (t - J2000) \times \dot{M}_{\odot},$$

$$M_P(t) = M_P,$$

$$G(t) = G(J2000) + (t - J2000) \times \dot{G},$$

$$\mu(t) = G(t) \times M(t),$$

- To obtain $\frac{\dot{G}}{G}$, one has to suppose $\frac{\dot{M}_{\odot}}{M_{\odot}}$
- Hypothesis on mass gain (comet, dust...) and on mass loss (solar wind...)

Reference	PE	Method	$\frac{\dot{\mu}_{\odot}}{\mu_{\odot}}$ $\times 10^{14} \text{ yr}^{-1}$	$\frac{\dot{M}_{\odot}}{M_{\odot}}$ $\times 10^{14} \text{ yr}^{-1}$	$\dot{G}/G$ $\times 10^{14} \text{ yr}^{-1}$
Before AU fixed					
Pitjeva (1993)	EPM1988	FF	NA	NA	470 ± 470
Pitjeva (2001)	EPM2000	FF	NA	NA	4 ± 8
Standish (2001)	DE405	FF	NA	NA	1 ± 8
Pitjeva (2005b)	EPM2004	FF	NA	NA	-2 ± 5
After AU fixed					
Konopliv et al. (2011a)	DE421	PF	1 ± 16	-9.2 ± 6.1	$10.2 \pm 22.1^*$
Pitjeva and Pitjev (2012)	EPM2010	FF	-5 ± 4	-6.7 ± 3.1	1.65 ± 8.77
Pitjeva and Pitjev (2013)	EPM2011	FF	-6.3 ± 6.4	-6.7 ± 3.1	0.4 ± 11.1
Fienga et al. (2015)	INPOP15a	FF	-5.0 ± 2.9	-9.2 ± 6.1	4.2 ± 9.0
		MC	-4.3 ± 7.4	-9.2 ± 6.1	4.9 ± 13.5
Genova et al. (2018)		MSG	-6.130 ± 1.47	-10 ± 1	4 ± 7.5
Pitjeva et al. (2021)	EPM2021	FF	-10.2 ± 1.4	-11.05 ± 2.35	0.85 ± 3.75
Fienga et al. (2021a)	INPOP20a	PF**	-8.8 ± 2.9	-9.2 ± 6.1	0.4 ± 9.0
				-11.05 ± 2.35	2.25 ± 5.25
		MC	-10.3 ± 22.8	-9.2 ± 6.1	-0.8 ± 28.4

(Pitjeva et al. 2021)

$$\frac{\dot{\mu}_{\odot}}{\mu_{\odot}} = (-10.2 \pm 1.4) \times 10^{-14} \text{ yr}^{-1}$$

$$\frac{\dot{G}}{G} = (0.85 \pm 3.75) \times 10^{-14} \text{ yr}^{-1}$$

Massive graviton / Yukawa potential / fifth force

W18: (Will 2018) ; B19: (Bernus et al. 2019)

Yukawa suppression of massive interactions at the Compton length, λ_g <i>What about if metric field has a mass, m_g ?</i>	$w = w_N \exp\left(-\frac{r}{\lambda_g}\right)$ (W18) $w \sim w_N \left(1 + \frac{1}{2} \frac{r^2}{\lambda_g^2}\right)$ (B19)	$\lambda_g = \frac{\hbar}{c m_g}$ with m_g the graviton mass
Fifth Force : α , the strenght of the force relative to gravity ; λ , the range of the force <i>What about if metric field doesn't have a mass but gravitational field does ?</i>	$w = w_N \left(1 + \alpha \exp\left(-\frac{r}{\lambda}\right)\right)$	If $0 < \alpha$ and $\lambda \gg r$, then $\lambda_g \approx \frac{\lambda}{\sqrt{ \alpha }}$

Modify EOM/TS/LP in PLE (Bernus, 2019, 2021, Mariani et al. 2023)

$$\delta \mathbf{a}_A^{\lambda_g} = \frac{1}{2\lambda_g^2} \sum_{A \neq T} \frac{\mu_T}{r_{AT}} \mathbf{r}_{AT} + \mathcal{O}(\lambda_g^{-3}),$$

$$c(t_r - t_e) = c(t_r - t_e)_{GRT} + \sum_A \frac{\mu_A}{c^2} \frac{1}{2\lambda_g^2} \ln \left[b^2 \frac{\mathbf{n} \cdot \mathbf{r}_{rA} + r_{rA}}{\mathbf{n} \cdot \mathbf{r}_{eA} + r_{eA}} + \mathbf{n} \cdot (r_{rA} \mathbf{r}_{rA} - r_{eA} \mathbf{r}_{eA}) \right]$$

Massive graviton in PE : example of correlated mis-interpretation

(Will 2018): postfit analysis *

(*) only λ_g explains the differences between ephemeris and observations

$$\Delta\varpi = \pi \left(\frac{a}{\lambda_g}\right)^2 (1 - e^2)^{-1/2},$$

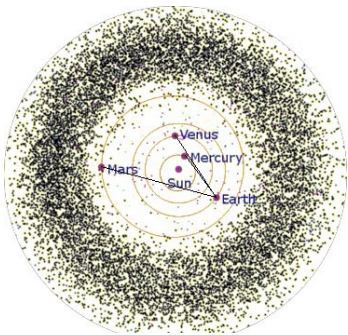
Table 1. Bounds on λ_g .

Planet	$\sigma(\dot{\varpi})$ (mas yr ⁻¹)	λ_g bound (10 ¹⁴ km)
Data from table 4 of [16]		
Mercury	0.03	0.18
Venus	0.016	0.28
Earth	0.0019	0.88
Mars	0.00037	2.21
Jupiter	0.28	0.11
Saturn	0.0047	0.98
Data from [17]		
Mercury	0.02	0.22
Saturn	0.026	0.42

(Will 2018)

- Of course , λ_g can not explain ALL the differences between ephemeris and observations

- Correlations between (> 400) parameters



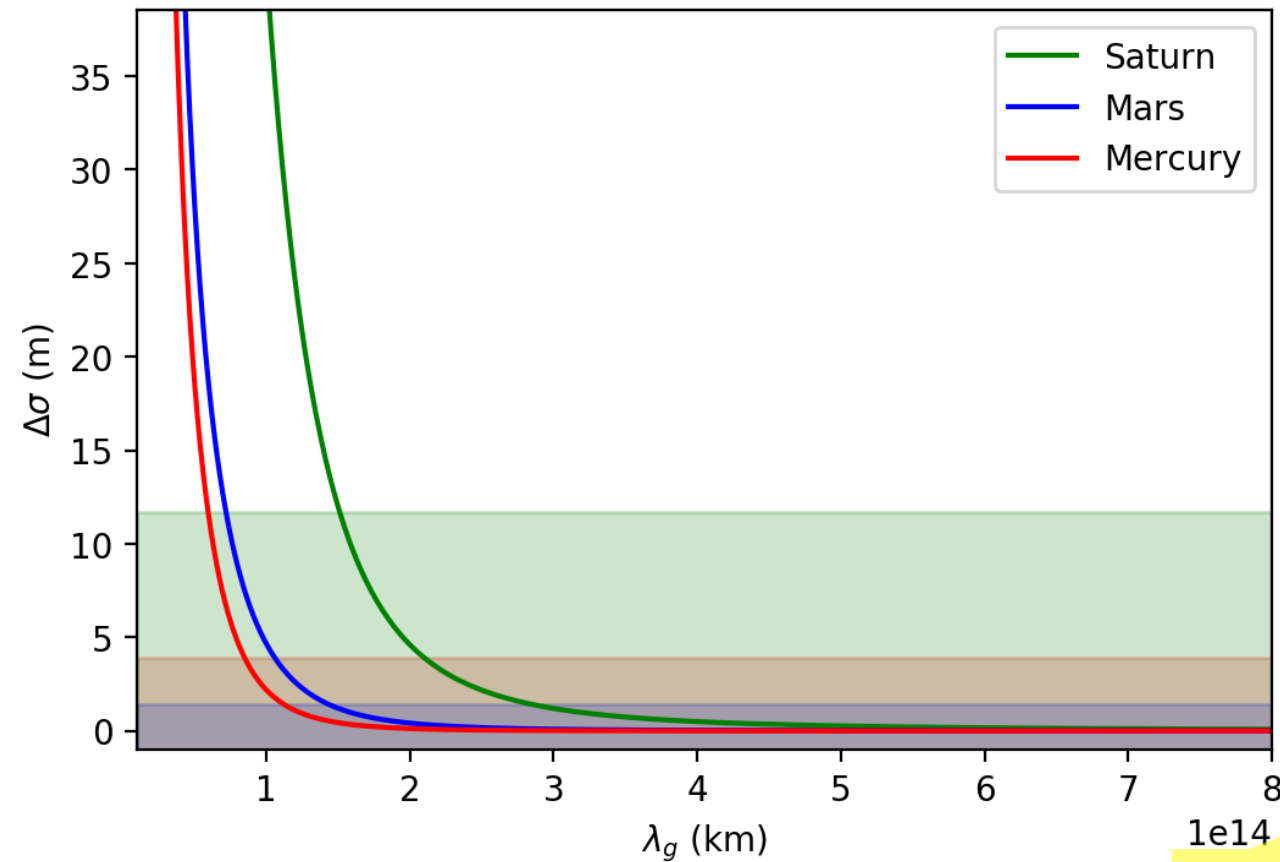
$$\Delta\dot{\varpi}_{PLA} = \frac{(2\gamma - \beta + 2)GM_{\odot}(t)}{a(1 - e^2)c^2} + \Delta\dot{\varpi}_{J_2^{\odot}}(J_2^{\odot}, a^2) + \Delta\dot{\varpi}_{AST}$$

	λ_g	a Mercury	a Mars	a Saturn	a Venus	a EMB	$GM_{\odot}$
λ_g	1	0.50	0.49	0.04	0.39	0.05	0.66
a Mercure	...	1	0.21	0.001	0.97	0.82	0.96
a Mars	...	...	1	0.03	0.29	0.53	0.06
a Saturn	...	...	...	1	0.003	0.02	0.01
a Venus	...	...	...	...	1	0.86	0.94
a EMB	...	...	...	...	...	1	0.73
$GM_{\odot}$	...	...	...	...	...	...	1

(Bernus et al. 2019)

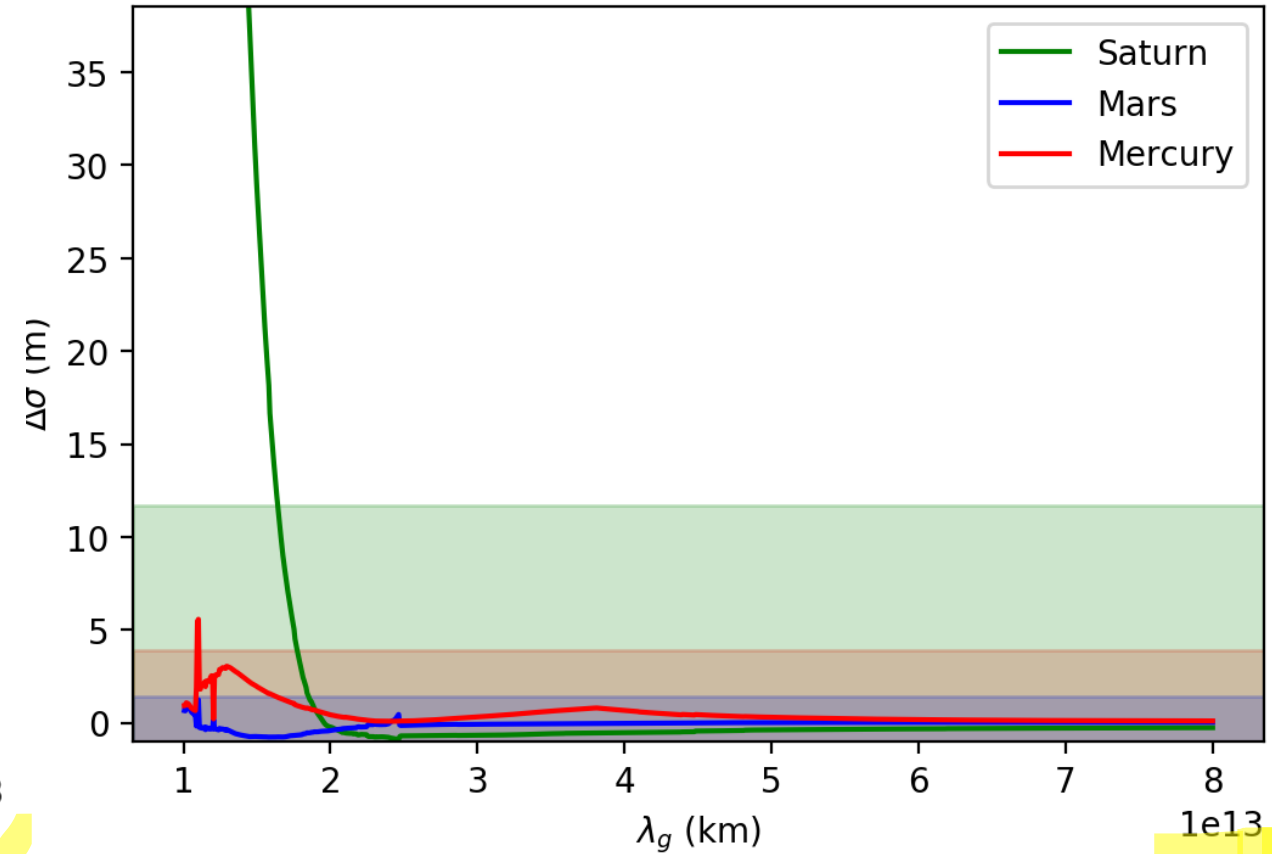
BEFORE FIT

Reference: INPOP17b



AFTER FIT of all parameters: IC, Sun mass, J2 Sun, GM_asteroids

Reference: INPOP17b

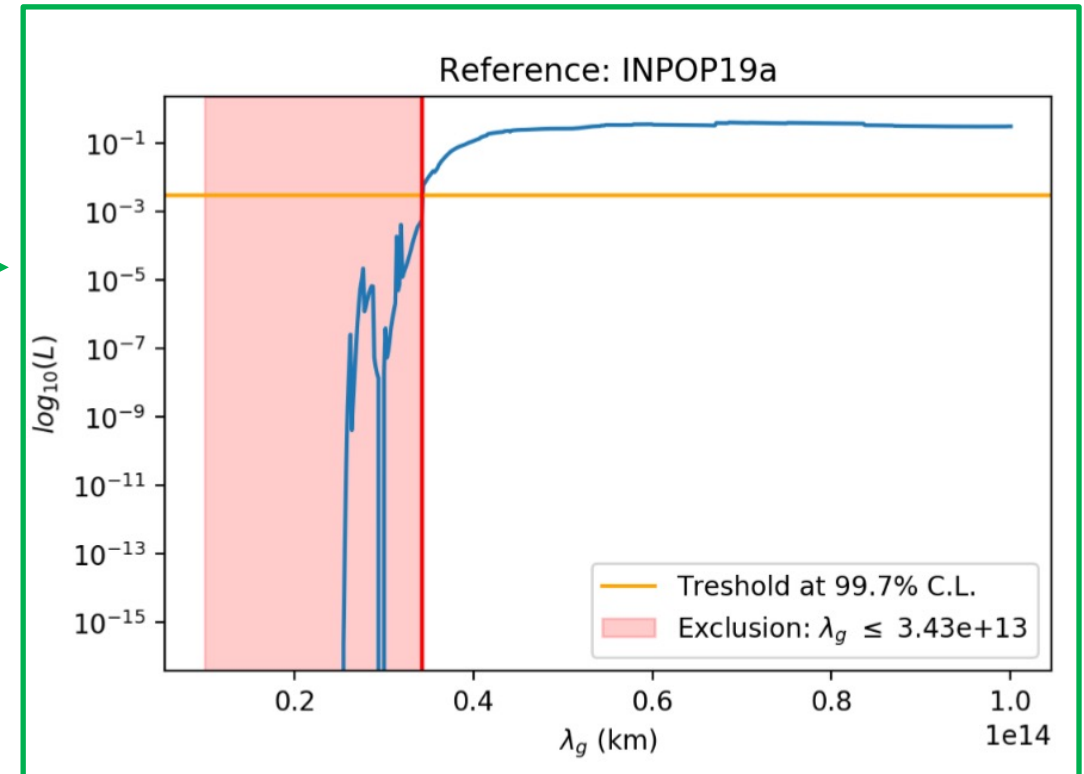


PLANETARY MASSES ARE ALWAYS FIXED

Massive graviton in PE : example of correlated mis-interpretation

(Bernus et al. 2019, 2021)

- Fixed $\lambda_g \rightarrow$ INPOP fully built with λ_g
- Random exploration of λ_g + cost function



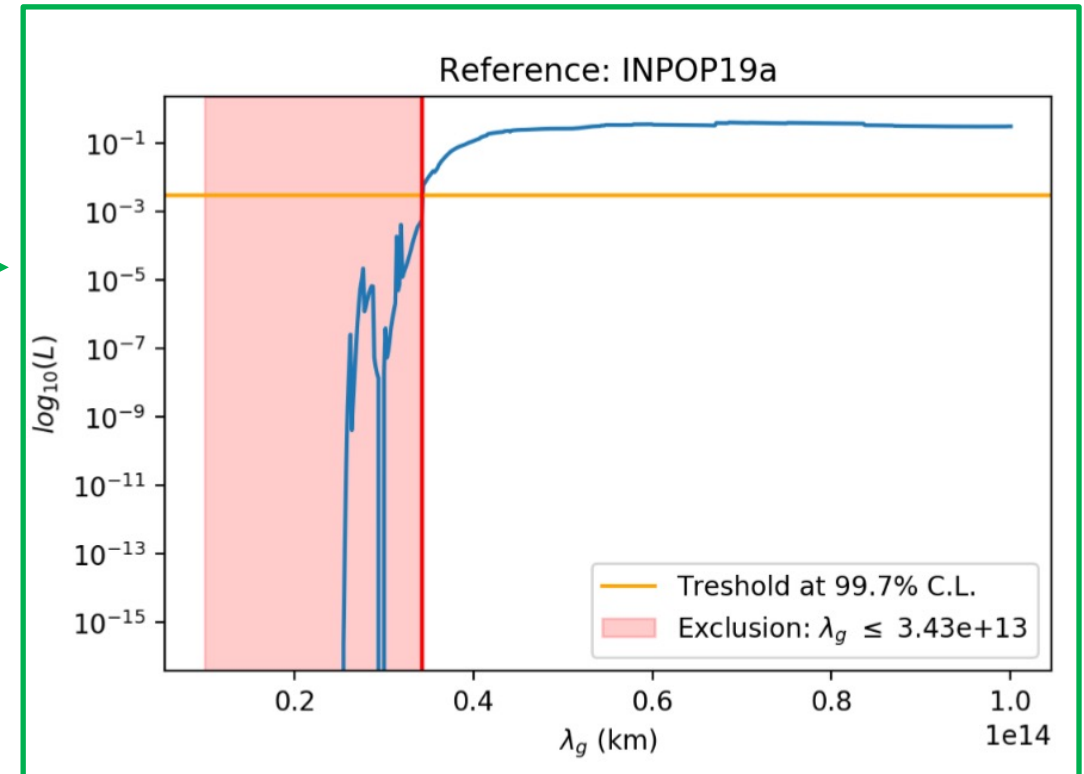
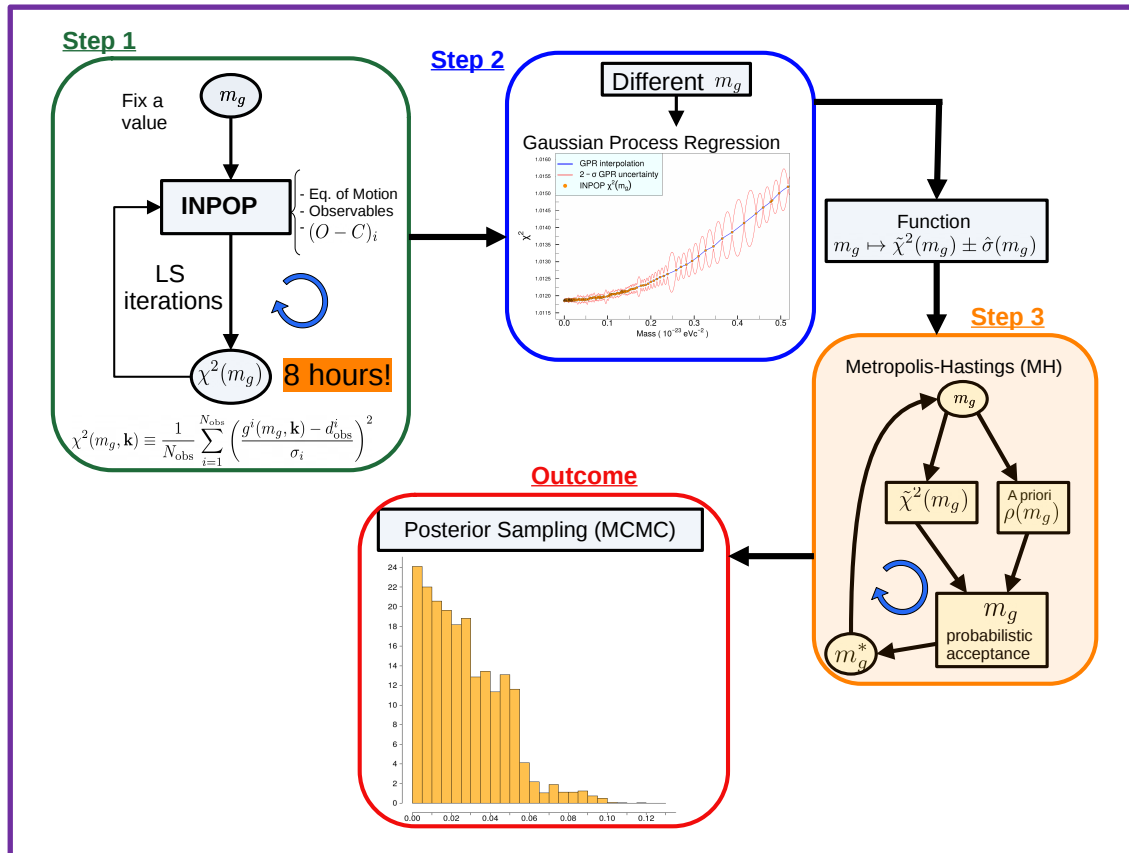
- 1 PLE iteration (EoM integration + fit) over 100 yrs = 5 mn
- From 400 to 800 parameters estimated per solutions
- Too long for MCMC for the full 400 parameters

Massive graviton in PE : example of correlated mis-interpretation

(Bernus et al. 2019, 2021)

- Fixed $\lambda_g \rightarrow$ INPOP fully built with λ_g
- Random exploration of λ_g + cost function

(Mariani et al. 2023) GPR+ MCMC



- 1 PLE iteration (EoM integration + fit) over 100 yrs = 5 mn
- From 400 to 800 parameters estimated per solutions
- Too long for MCMC for the full 400 parameters

$\rightarrow$ GPR on several hundreds of runs and then MH

Massive graviton / Yukawa suppression

(Bernus et al., 2019, 2021) (Mariani et al., 2023)

	GWTC-1	GWTC-3	Random exploration (RW) + cost function		MCMC	
CL	0.90	0.90	INPOP17a	INPOP19a	INPOP21a RW	INPOP21a MCMC
Graviton mass						
$\lambda_g \times 10^{-13}$ [km]	2.6	9.77	1.83	3.93	12.01	209.67
$m_g \times 10^{23}$ [eV/c ²]	4.7	1.27	6.76	3.16	1.03	0.059
Fifth Force						
$\frac{\lambda}{\sqrt{ \alpha }} \times 10^{-13}$ [km], $\alpha > 0$			1.83	3.93		
$\frac{\lambda}{\sqrt{ \alpha }} \times 10^{-13}$ [km], $\alpha < 0$				3.77		

(Mariani et al., 2023)

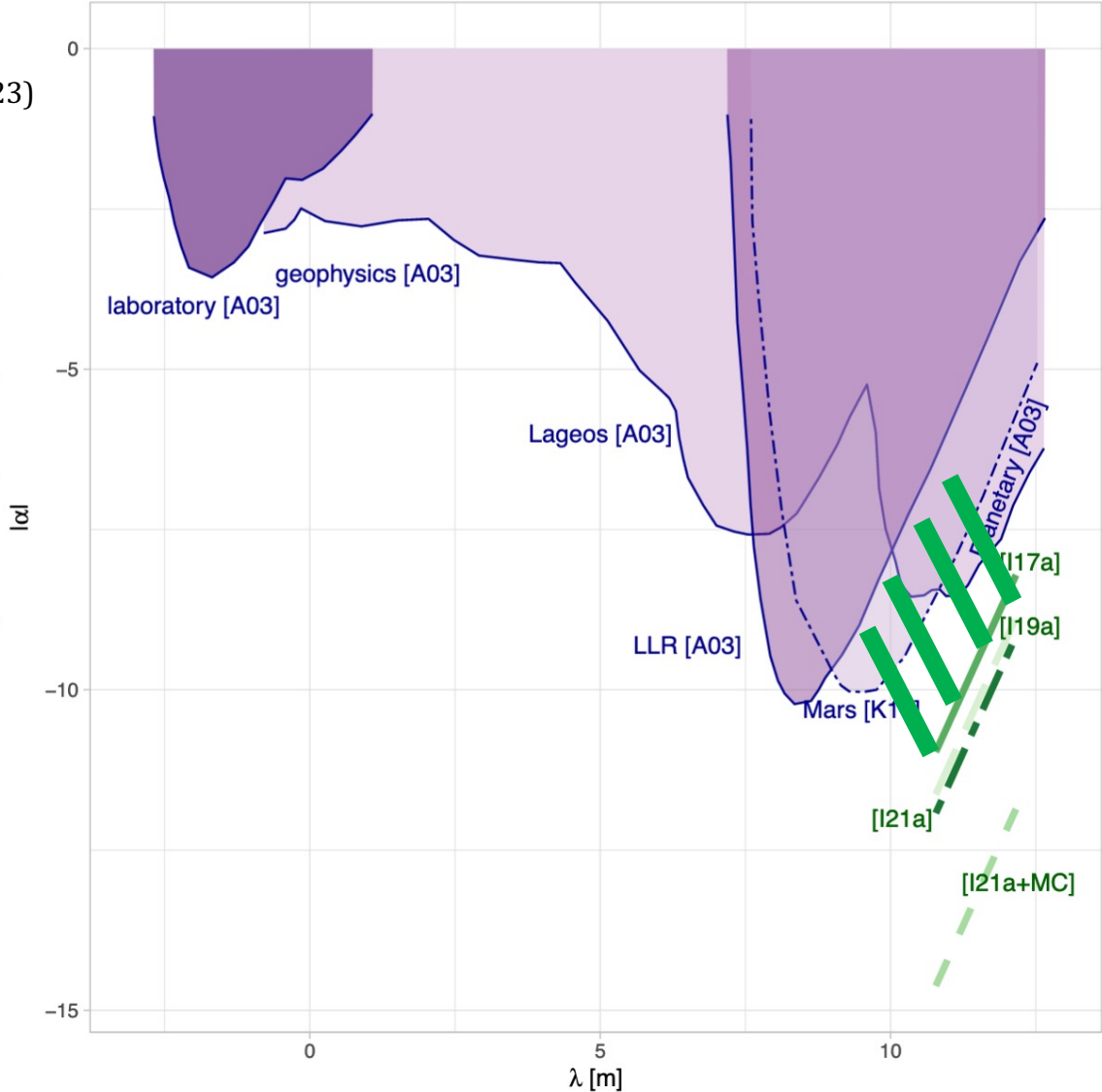
$$m_g < 0.10 \times 10^{-23} \text{ eV/c}^2. \text{ C.L. 99.7 \%}$$

$$\lambda_g > 122.48 \times 10^{13} \text{ km C.L. 99.7 \%}$$

(Will 2018):
postfit analysis *

$$\lambda_g > 1.4 - 2.7 \times 10^{14} \text{ km}$$

$$m_g < 0.40 - 0.8 \times 10^{-23} \text{ eV/c}^2.$$



(Mariani et al., Bayesian test of the mass of the graviton with planetary ephemerides. Phys.Rev. D, 108:024047, Jul 2023)

The Brans-Dicke scalar-field / Einstein dilaton theories.

In this type of theories, one introduces a scalar field φ and its kinematic term $f(\varphi)$ such that we can derive from the metric tensor g , a other metric g^* such as

$$g_{\mu\nu} = \frac{f_0}{f(\varphi)} g_{\mu\nu}^*, \quad (139a)$$

and the mass of the particule A, m_A , is then

$$m_A^*(\phi) = \sqrt{\frac{f_0}{f(\varphi)}} m_A(\varphi).$$

→ Matter is then coupled with the scalar field. This **coupling** can be universal (i.e BD) or matter-dependent (dilaton) with Z

$$\left(\frac{d\phi}{d\varphi}\right)^2 = \frac{Z(\varphi)}{2} = \frac{\omega(\varphi)}{\varphi f(\varphi)} + \frac{3}{2} \left(\frac{f'(\varphi)}{f(\varphi)}\right)^2,$$

In this context, the new metric can written with

$$g_{00}^* = -1 + 2\frac{w_*}{c^2} - 2\frac{w_*^2}{c^4} + O(c^{-6}),$$

$$g_{0i}^* = -4\frac{w_*^i}{c^3} + O(c^{-5}),$$

$$g_{ij}^* = \delta_{ij} \left(1 + 2\frac{w_*}{c^2}\right) + O(c^{-4}),$$

$$w_* = w_{0*} - \frac{1}{c^2} \Delta_* + O(c^{-4}),$$

$$w_*^i = \sum_A \frac{G_* m_{A0}}{r_A} v_A^i + O(c^{-2}),$$

$$w_0^* = \sum_A \frac{G_* m_{A0}}{r_A},$$

$$\Delta_* = \sum_A \frac{G_* m_{A0}}{r_A} \left[\frac{\mathbf{r}_A \cdot \mathbf{a}_A}{2} + \frac{(\mathbf{r}_A \cdot \mathbf{v}_A)^2}{2r_A^2} - 2v_A^2 \right. \\ \left. + \sum_{B \neq A} (1 + \alpha_{A0}^* \alpha_{B0}^*) \frac{G_* m_{B0}}{r_{AB}} \right], \quad (140c)$$

Einstein massless dilaton : INPOP test (Bernus et al. 2022)

- Generic formalism allowing both WEP, GWEP violations
- Non-universal coupling between scalar field φ and matter
- Parameters of the metric depend on Z (dilaton) or universal constant (Brans-Dicke theory, BDT)
- Effect on light path with γ and negligible additional terms

$$g_{\mu\nu} = \frac{f_0}{f(\varphi)} g_{\mu\nu}^*, \quad m_A^*(\phi) = \sqrt{\frac{f_0}{f(\varphi)}} m_A(\varphi).$$

$$\delta_A = \frac{\alpha_0 \tilde{\alpha}_A}{1 + \alpha_0^2}, \quad \delta_{AB} = \frac{\tilde{\alpha}_A \tilde{\alpha}_B}{1 + \alpha_0^2},$$

$$\mu_A = \frac{G}{f_0} (1 + \alpha_0^2) (1 + \delta_A) m_A(\varphi_0),$$

$$\gamma = \frac{1 - \alpha_0^2}{1 + \alpha_0^2}, \quad \beta = 1 + \frac{\beta_0(Z)}{2} \frac{\alpha_0^2}{(1 + \alpha_0^2)^2},$$

$$\begin{aligned} \mathbf{a}_T = & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3} \mathbf{r}_{AT} (1 + \delta_T + \delta_{AT}) - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3 c^2} \mathbf{r}_{AT} \left\{ \gamma v_T^2 + (\gamma + 1) v_A^2 \right. \\ & - 2(1 + \gamma) \mathbf{v}_A \cdot \mathbf{v}_T - \frac{3}{2} \left(\frac{\mathbf{r}_{AT} \cdot \mathbf{v}_A}{r_{AT}} \right)^2 - \frac{1}{2} \mathbf{r}_{AT} \cdot \mathbf{a}_A \\ & \left. - 2\gamma \sum_{B \neq T} \frac{\mu_B}{r_{TB}} + \sum_{B \neq A} \frac{\mu_B}{r_{AB}} \right\} + \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} [2(1 + \gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_T - \\ & (1 + 2\gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_A] (\mathbf{v}_T - \mathbf{v}_A) + \frac{3+4\gamma}{2} \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} \mathbf{a}_A \end{aligned}$$

Einstein massless dilaton : INPOP test (Bernus et al. 2022)

- Generic formalism allowing both WEP, GWEP and SEP violation
- Non-universal coupling between scalar field and matter
- Parameters of the metric can depend on Z

$$g_{\mu\nu} = \frac{f_0}{f(\varphi)} g_{\mu\nu}^*,$$

$$\delta_A = \frac{\alpha_0 \tilde{\alpha}_A}{1 + \alpha_0^2}, \quad \delta_{AB} = \frac{\tilde{\alpha}_A \tilde{\alpha}_B}{1 + \alpha_0^2},$$

$$\mu_A = \frac{G}{f_0} (1 + \alpha_0^2) (1 + \delta_A) m_A(\varphi_0),$$

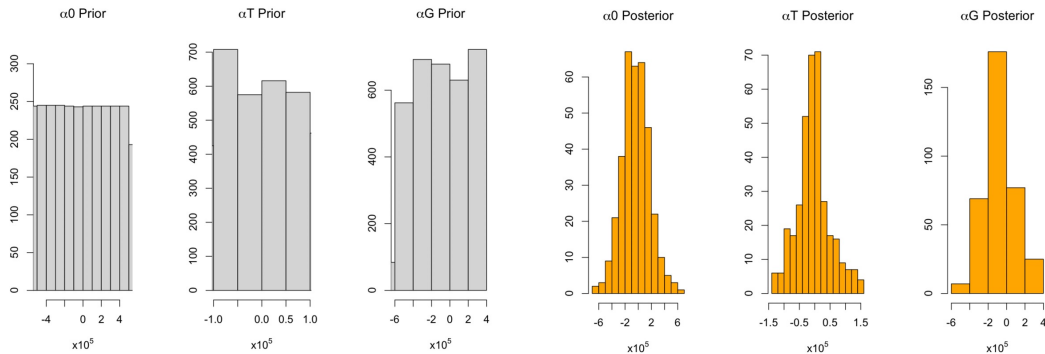
$$\gamma = \frac{1 - \alpha_0^2}{1 + \alpha_0^2}, \quad \beta = 1 + \frac{\beta_0(Z)}{2} \frac{\alpha_0^2}{(1 + \alpha_0^2)^2},$$

(Bernus et al. 2022)

- Introduction in PE of previous EIHDL and Shapiro modified equations
- Linear coupling ($\beta_0 = 0, \beta = 1$)
- Random exploration for α_G, α_T and α_0 starting with flat large priors
- Cost functions (same approach as for (Bernus et al. 2021) graviton)

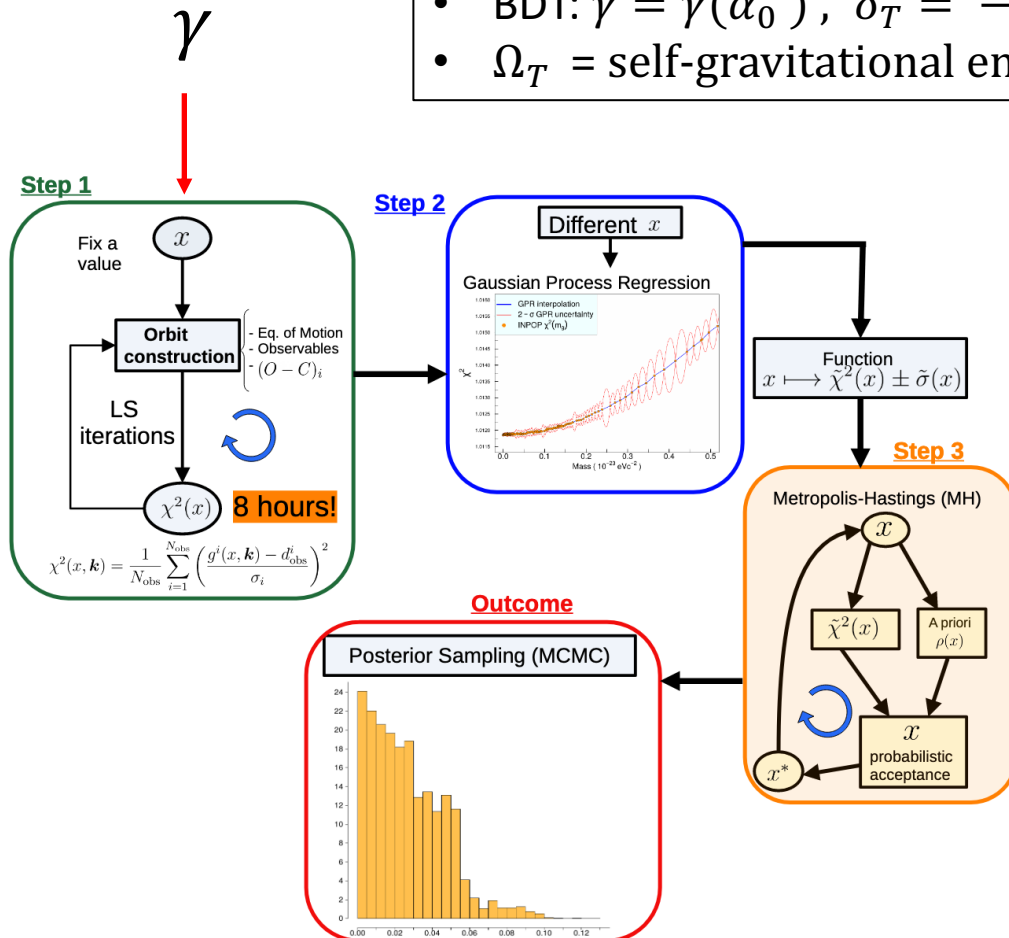
- α_0 : universal coupling
- α_T : Telluric planet (C,S)
- α_G : Gazeous planet (H,He)

Confidence:	90%	99.5%
$\alpha_0 (\times 10^5)$	-0.94 ± 5.35	1.01 ± 23.7
$\alpha_T (\times 10^6)$	0.24 ± 1.62	0.00 ± 24.5
$\alpha_G (\times 10^5)$	0.01 ± 4.38	-1.46 ± 12.0
$(\gamma - 1) \times 10^8$	0.2 ± 6	0.2 ± 11.2



Brans-Dicke Theory = Einstein-dilaton with universal coupling only

- Introduction in INPOP of previous EIHDL and Shapiro modified equations
- Linear coupling ($\beta_0 = 0, \beta = 1$)
- Only the universal coupling, α_0
- BDT: $\gamma = \gamma(\alpha_0)$, $\delta_T = -(1 - \gamma(\alpha_0)) \times \Omega_T$, $\delta_{AT} = 0$
- Ω_T = self-gravitational energy $\rightarrow \delta_T = \eta \times \Omega_T$



$$\begin{aligned} \mathbf{a}_T = & - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3} \mathbf{r}_{AT} (1 + \delta_T + \delta_{AT}) - \sum_{A \neq T} \frac{\mu_A}{r_{AT}^3 c^2} \mathbf{r}_{AT} \left\{ \gamma v_T^2 + (\gamma + 1) v_A^2 \right. \\ & \left. - 2(1 + \gamma) \mathbf{v}_A \cdot \mathbf{v}_T - \frac{3}{2} \left(\frac{\mathbf{r}_{AT} \cdot \mathbf{v}_A}{r_{AT}} \right)^2 - \frac{1}{2} \mathbf{r}_{AT} \cdot \mathbf{a}_A \right. \\ & \left. - 2\gamma \sum_{B \neq T} \frac{\mu_B}{r_{TB}} + \sum_{B \neq A} \frac{\mu_B}{r_{AB}} \right\} + \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} [2(1 + \gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_T - \\ & (1 + 2\gamma) \mathbf{r}_{AT} \cdot \mathbf{v}_A] (\mathbf{v}_T - \mathbf{v}_A) + \frac{3+4\gamma}{2} \sum_{A \neq T} \frac{\mu_A}{c^2 r_{AT}^3} \mathbf{a}_A \end{aligned}$$

$$\gamma = \frac{1 - \alpha_0^2}{1 + \alpha_0^2}$$

$$\eta = \gamma - 1 = -2 \frac{\alpha_0^2}{1 + \alpha_0^2}$$

Direct tie between SEP violation and $\gamma > 0$

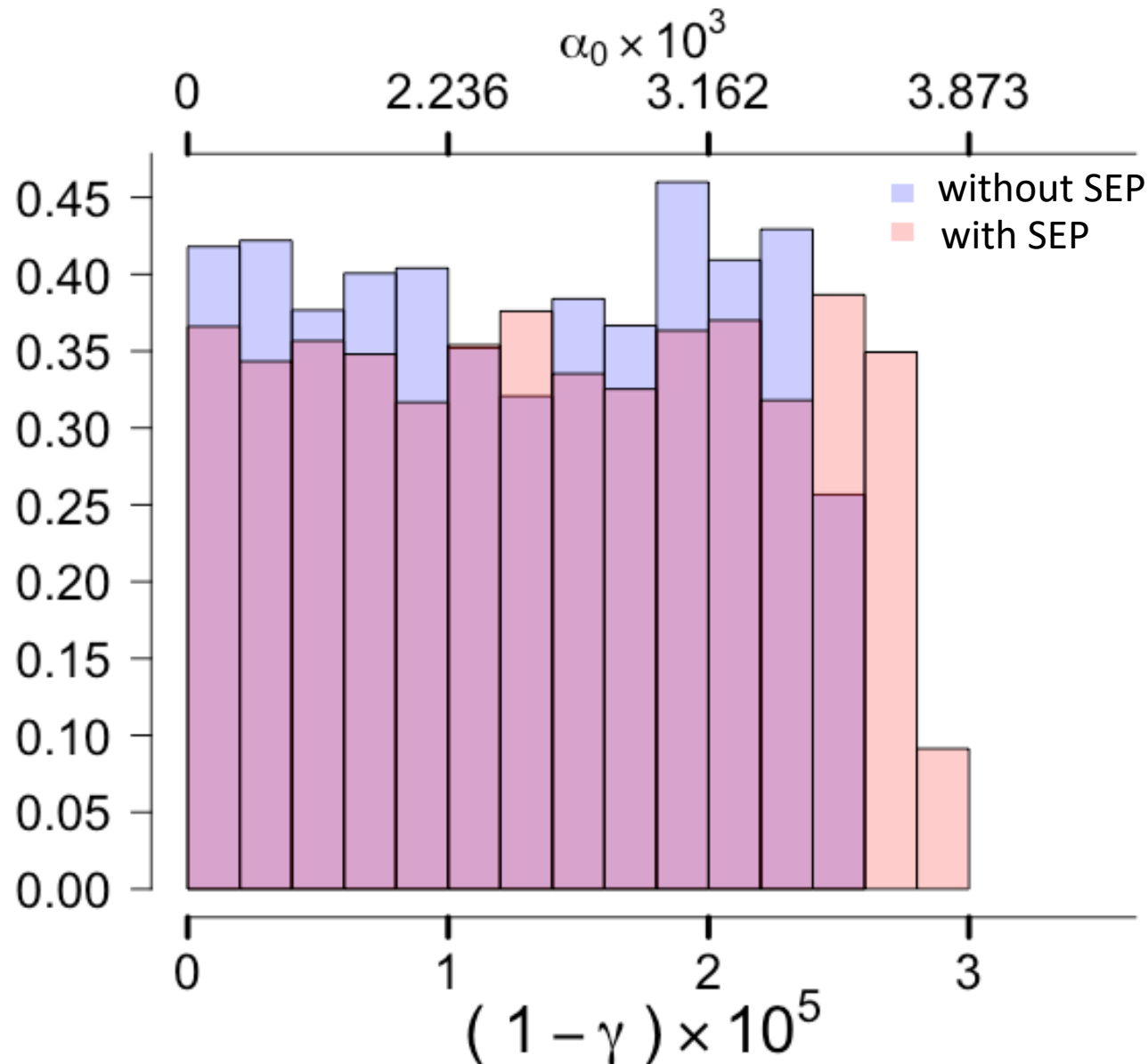
Brans-Dicke Theory ($\alpha_T = \alpha_G = 0$) and GPR+MCMC (Mariani et al. 2024)

Case 1: γ wo SEP
 ($\beta = 1, \alpha_0 = \eta = 0$)

$(1 - \gamma) \leq 2.5 \times 10^{-5}$
 @ 99.7 CL

Cassini conjunction test
 (Bertotti et 2003):
 $|1 - \gamma| \leq 4.4 \times 10^{-5}$

γ Uniform prior from
 $[0: 1.5 \times 10^{-4}]$



Case 2: BDT γ with SEP
 violation ($\beta = 1, \alpha_0 \neq 0$)

$\gamma = \frac{1 - \alpha_0^2}{1 + \alpha_0^2}; \eta = -2 \frac{\alpha_0^2}{1 + \alpha_0^2}$

$(1 - \gamma) \leq 2.83 \times 10^{-5}$
 $\alpha_0 \leq 3.76 \times 10^{-3}$

$|\eta| \leq 2.83 \times 10^{-5}$
 @ 99.7 CL

γ Uniform prior from
 $[0: 1.5 \times 10^{-4}]$

Direct versus indirect tests : example of dark matter

Direct Tests

A new dynamical modeling is developed in an alternative framework with equations of motions, time scales and light path described in this new framework (PPN, graviton, dilaton), adjustment included

PROS

Totally consistent results with good estimations of uncertainties and correlations

CONS

Quite heavy to developp

Indirect Tests

Interpretation of derived quantities (advance of perihelia) in terms of constraints

Theories	Section	Impact on orbits	References
MOND	4.7	$d\ddot{\omega}_{supp}, d\dot{\Omega}_{supp}$	(Blanchet and Novak, 2011)
AWE/chameleons		variation of PPN parameters	(Füzfa and Alimi, 2007)
Scalar field theories		EP	(Burrage and Sakstein, 2018)
		$\dot{G}/G$	(Uzan, 2003)
dark Energy		variation of of PPN parameters	
dark Matter	5.2	$\dot{G}/G$	(Steinhardt and Wesley, 2009)
		linear drift of AU	(Arakida, 2010)
		a_{supp}	(Nordtvedt, 1994)
		$d\ddot{\omega}_{supp}, d\dot{\Omega}_{supp}$	(Frère et al., 2008)
Yukawa, fifth force	4.5		(Merkowitz, 2010)
f(r)		$d\ddot{\omega}_{supp}$	(de Felice and Tsujikawa, 2010)
		a_{supp}	
Massive graviton	4.4	variation of of PPN parameters	
		$d\ddot{\omega}_{supp}, d\dot{\Omega}_{supp}$	(Will, 2018a)

PROS

Very fast and relatively easy

CONS

Highly inconsistent with clear under-estimation of uncertainties and overestimation of the constraints

Comments on dark matter constraints with PE

$$\Delta \ddot{\varpi}_{PLA} = \frac{(2\gamma - \beta + 2)GM_{\odot}(t)}{a(1 - e^2)c^2} + \Delta \ddot{\varpi}_{J_2^{\odot}}(J_2^{\odot}, a^2) + \Delta \ddot{\varpi}_{AST} + GR_{AV}, DILA...$$

		PROs	CONs	
supplementary advances of perihelia	Khriplovich and Pitjeva (2006)	$< 7 \cdot 10^{-11}$ solar mass	Postfit interpretation Not Consistent	$\Delta \varpi_A^{DM} = -3\pi \frac{\rho_{DM}}{M_{sun}} \sqrt{1 - e_A^2}$
dark matter falling into the Sun $\frac{\dot{M}_{\odot}}{M_{\odot}}$	Blennow et al. 2018, Kardashev et al. 2005	Consistent	Hypothesis on mass gain (comet, dust...) and on mass loss (solar wind...)	$\frac{\dot{\mu}_{\odot}}{\mu_{\odot}} = \frac{\dot{M}_{\odot}}{M_{\odot}} + \frac{\dot{G}}{G}$
Additional acceleration from DM out of SS	Belbruno and Green (2022)		Affected only objects $a > 80$ AU	
Additional acceleration from DM in SS	Pitjeva and Pitjev (2013)	Consistent	Too sensitive to mass distribution (P9 case)	$\mathbf{a}_A^{DM} = \frac{\mu_{M(R)}}{r_{AT}} \mathbf{r}_{AT}$

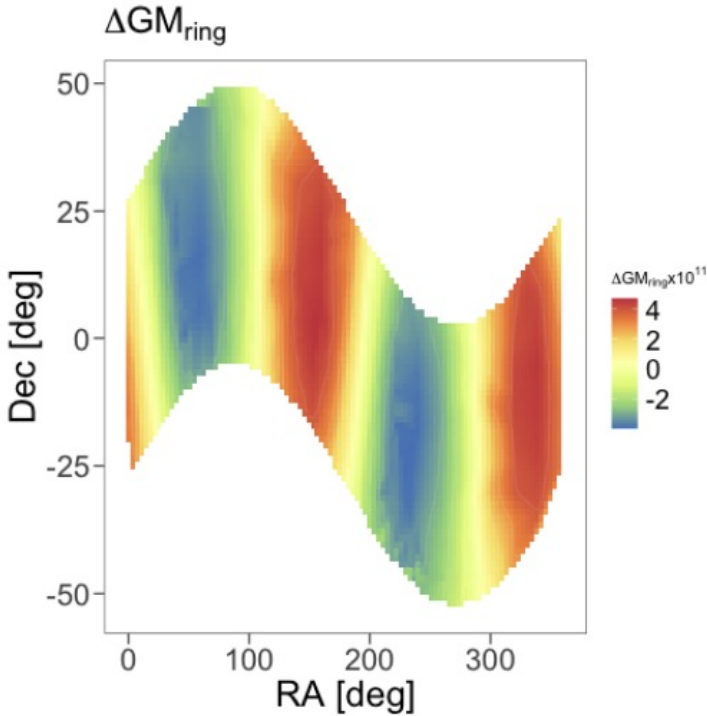
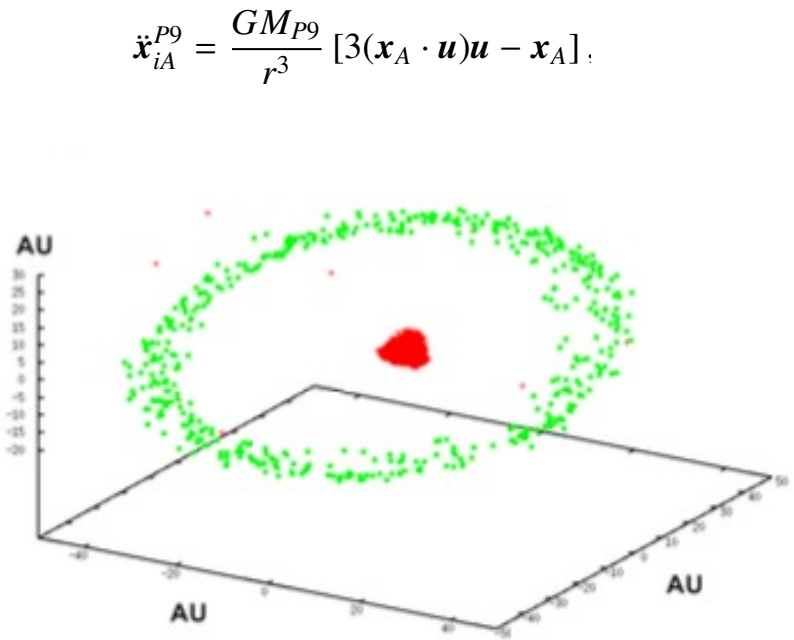
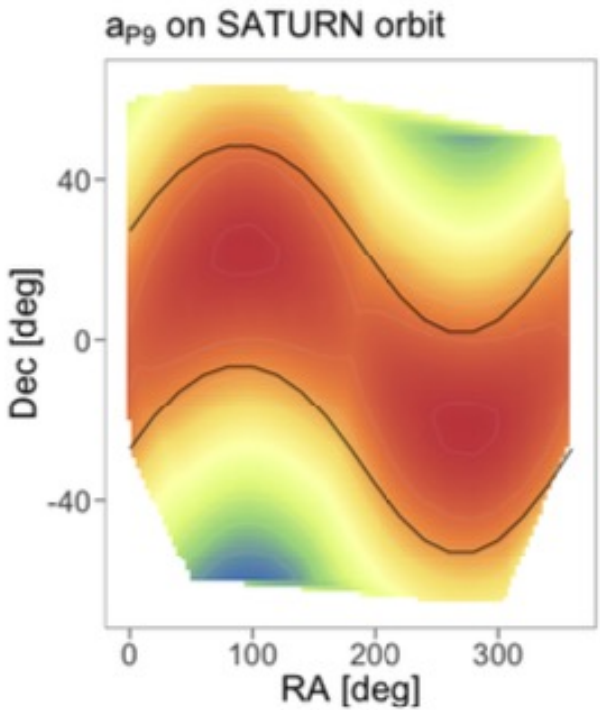
Detectability with ultralight dark matter ?

Comments on dark matter constraints with PE

$$\Delta \ddot{\omega}_{PLA} = \frac{(2\gamma - \beta + 2)GM_{\odot}(t)}{a(1 - e^2)c^2} + \Delta \ddot{\omega}_{J_2^{\odot}}(J_2^{\odot}, a^2) + \Delta \ddot{\omega}_{AST} + \textcolor{blue}{GRAV}, \textcolor{blue}{DILA}...$$

		PROs	CONs	
Additional acceleration from DM in SS	Pitjeva and Pitjev (2013)	Consistent	Too sensitive to mass distribution (P9 case)	$\textcolor{black}{a}_A^{DM} = \frac{\mu_{M(R)}}{r_{AT}} \textcolor{black}{r}_{AT}$

DM acceleration hidden by P9 acceleration hidden by KBO ring !!



Comments on dark matter constraint with LLR

(Zhang et al. 2022): Violation of EP for Galaxy DM

$$\Delta g_{EM} = a_M - a_E.$$
$$\Delta r_{EM} = A \cos(\omega(t - t_0) - \theta),$$
$$A = -\frac{3}{2} \frac{\Delta g_{EM}}{\omega(\omega - \omega_0)}.$$

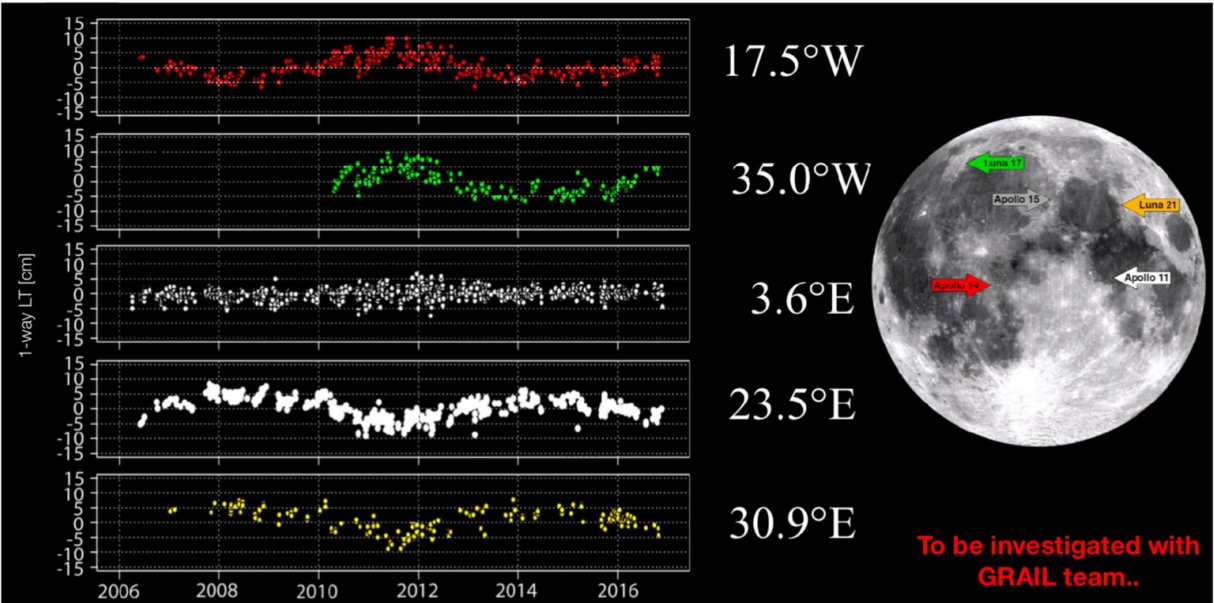
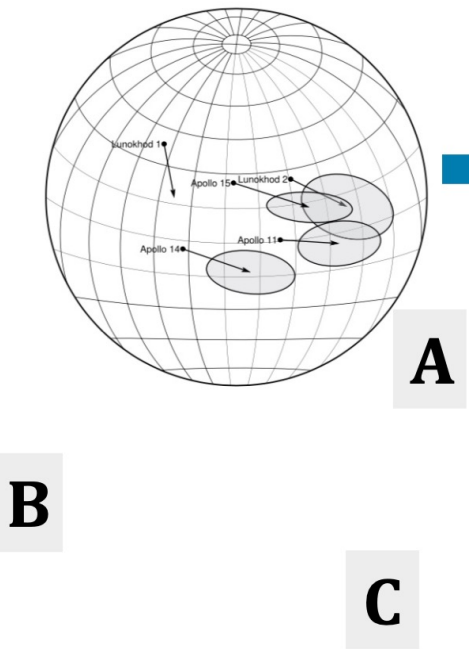
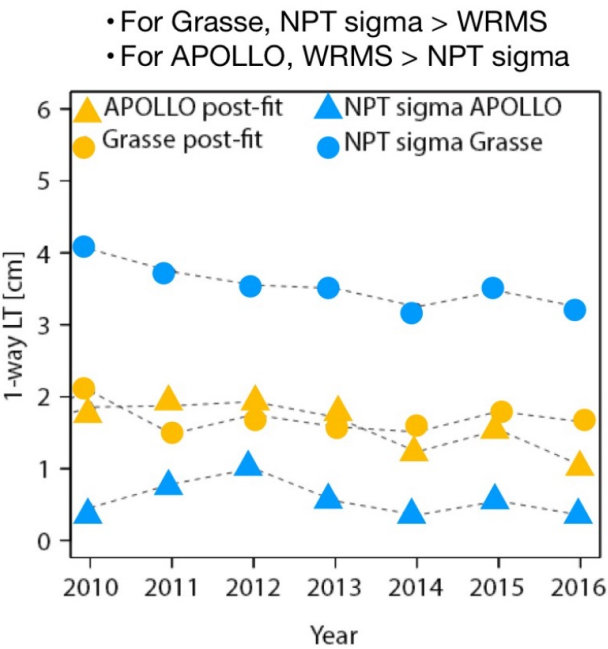
Station	Time period	Number of residuals	A (mm)
OCA	1984–1986	1206	-21.9 ± 6.2
	1987–1994	3237	6.6 ± 1.1
	1995–2018	10774	0.4 ± 0.2

Station	Time period	Number of residuals	A (mm)
APOLLO	2006–2016	2603	0.7 ± 0.4

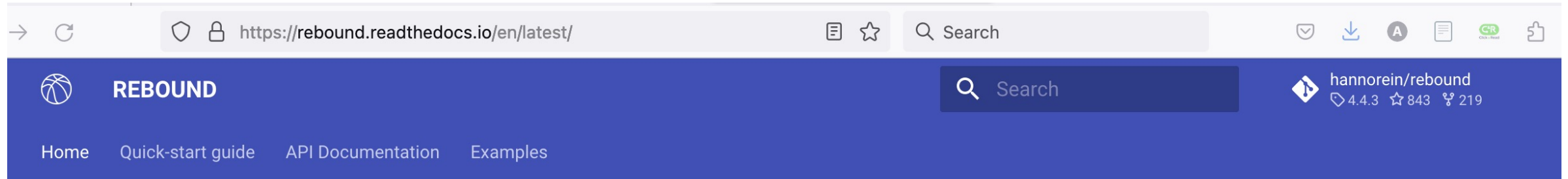
The given errors are 1σ standard deviations from the least-squares adjustment

(Zhang et al 2022) : No violation !

ω = lunar frequencies ; θ = direction of GC



Make your own tests



Home

Welcome to REBOUND

Features

Contributors

YouTube tutorials

Related projects

Additional physics

Analytical and semianalytical tools

Ephemeris-quality integrations of test particles

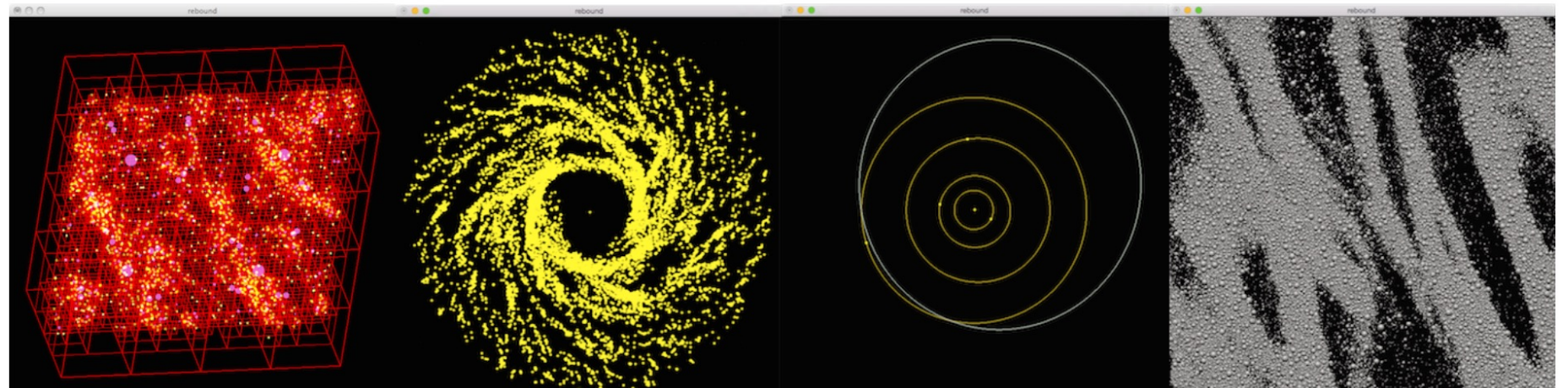
Papers

Acknowledgements

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Changelog

Welcome to REBOUND



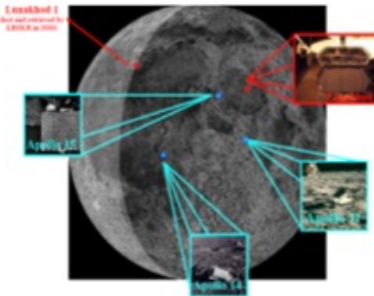
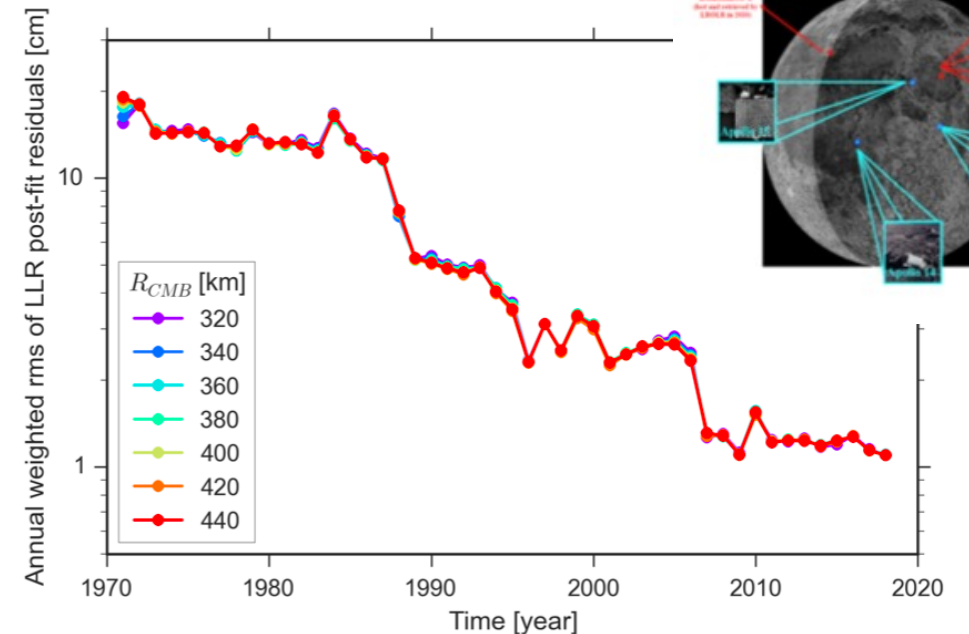
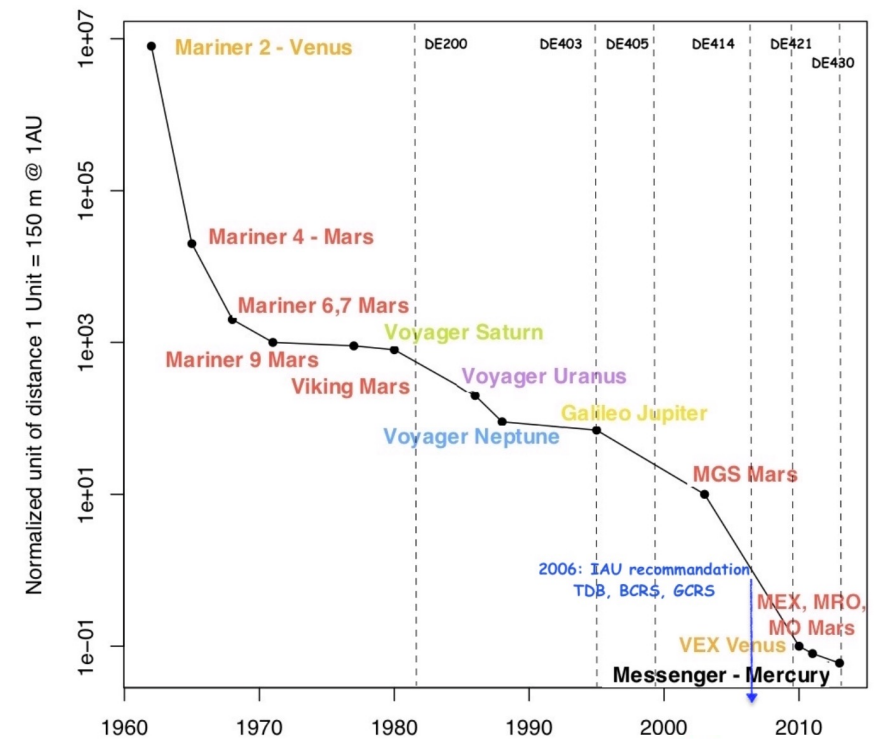
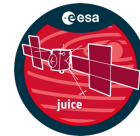
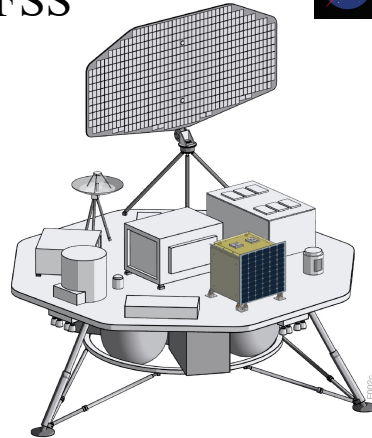
REBOUND is an N-body integrator, i.e. a software package that can integrate the motion of particles under the influence of gravity. The particles can represent stars, planets, moons, ring or dust particles. REBOUND is very flexible and can be customized to accurately and efficiently solve many problems in astrophysics.

Future developments

- **2026-2027 Mercury exploration with Bepi-Colombo (ESA/JAXA)**
- 2031-2022 Jupiter system with JUICE (ESA)
- NASA Art  mis program for Moon exploration : FSS (2026)
- ESA Moonlight (2028), Argonaut payload (post 2028)

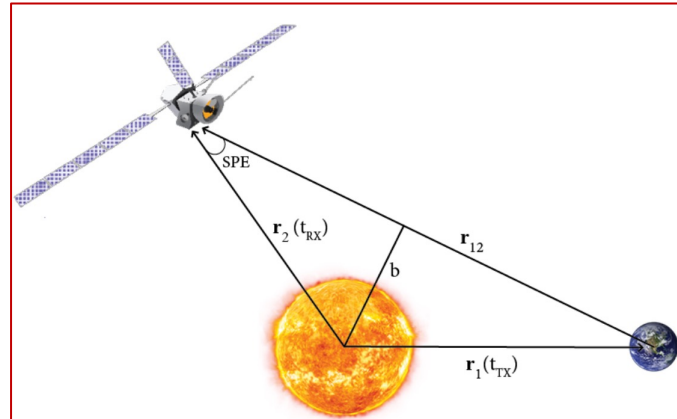
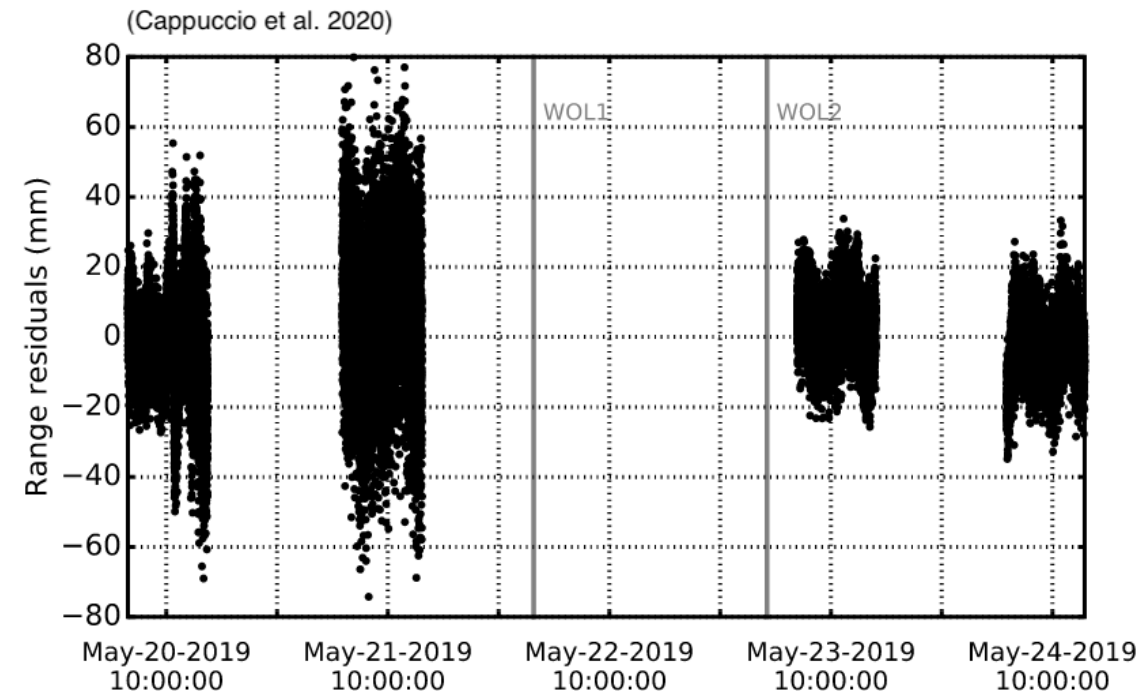
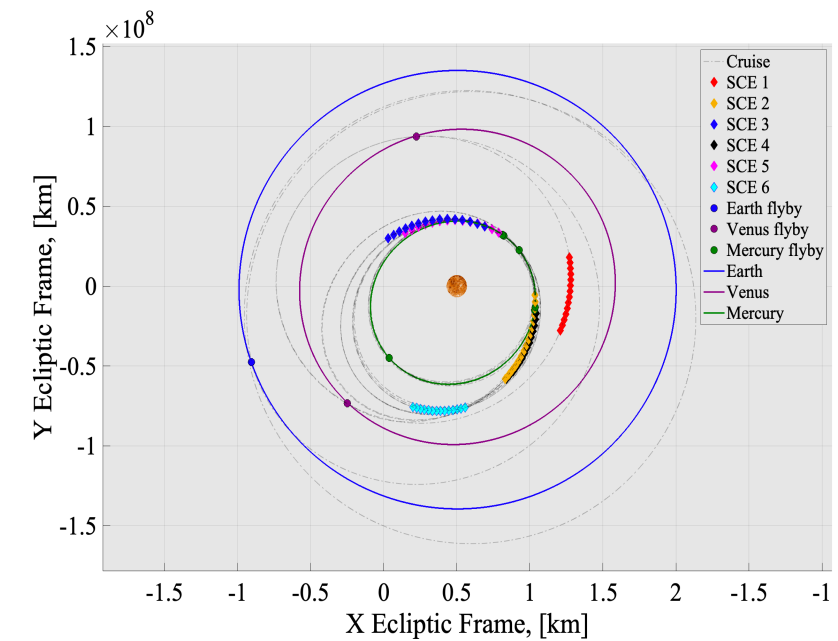
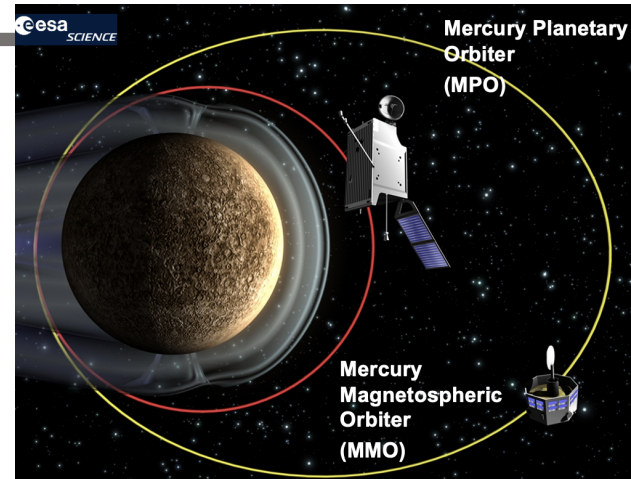


FSS



Bepi-Colombo: MORE experiment

- Exploration of Mercury starting in 2026
- Launched in 2018/10/20
- MORE = Mercury Orbit Radio Experiment
 - New technology Ka/Ka band
 - Less sensitive to solar plasma
 - Below cm accuracy on Mercury-Earth
- Objectives
 - Determination of Mercury gravity field
 - Characterisation of its inner structure
 - Tests of GRT and alternative theories
 - SCE → γ
 - Orbital phase



Testing Theories of Gravity with Planetary Ephemerides

Agnès Fienga, Olivier Minazzoli

<https://link.springer.com/article/10.1007/s41114-023-00047-0>

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