Testing gravity with planetary ephemerides

Part I (ex part II): Planetary ephemerides and gravity

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Overview

1) General introduction to the planetary and lunar ephemerides (PLE)
   - Introduction
   - GR framework
   - Accuracies

2) Tests in the PPN framework
   - Framework
   - Results and issues
   - Equivalence Principal

3) Other alternative theories
   - Time varying constants
   - Massive graviton, Yukawa suppression
   - Einstein Dilaton
   - Dark matter

4) Conclusions

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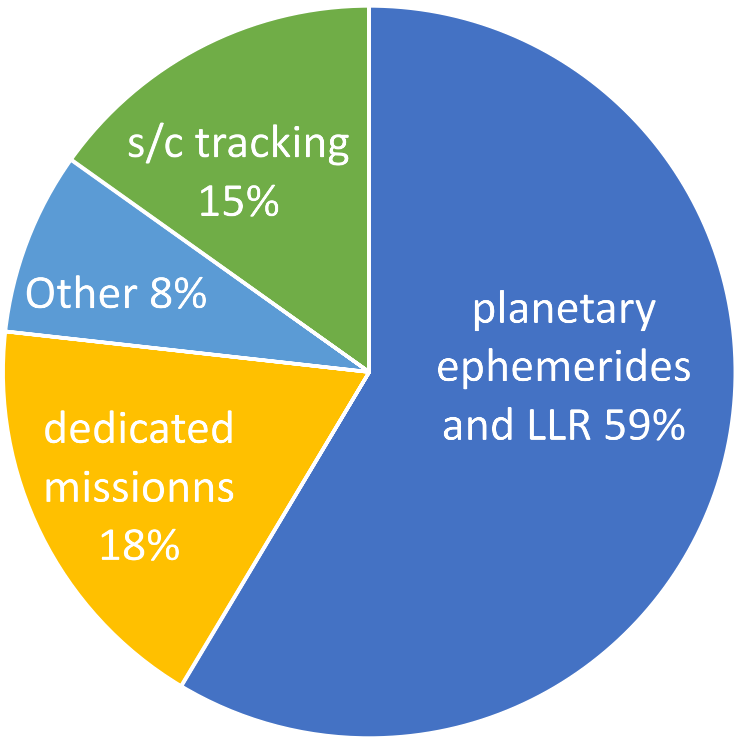
4) Conclusions

Why do we need tests of gravitation in the solar system ?

- 80 publications during the last 20 years
- Dedicated space-missions : GP-B, Microscope and Secondary goals of mission: Bepi-Colombo, Cassini, GNSS
- Theories can only be tested in weak-field (solar system) environment : screening mechanism
- solar system = boundary condition of cosmology !



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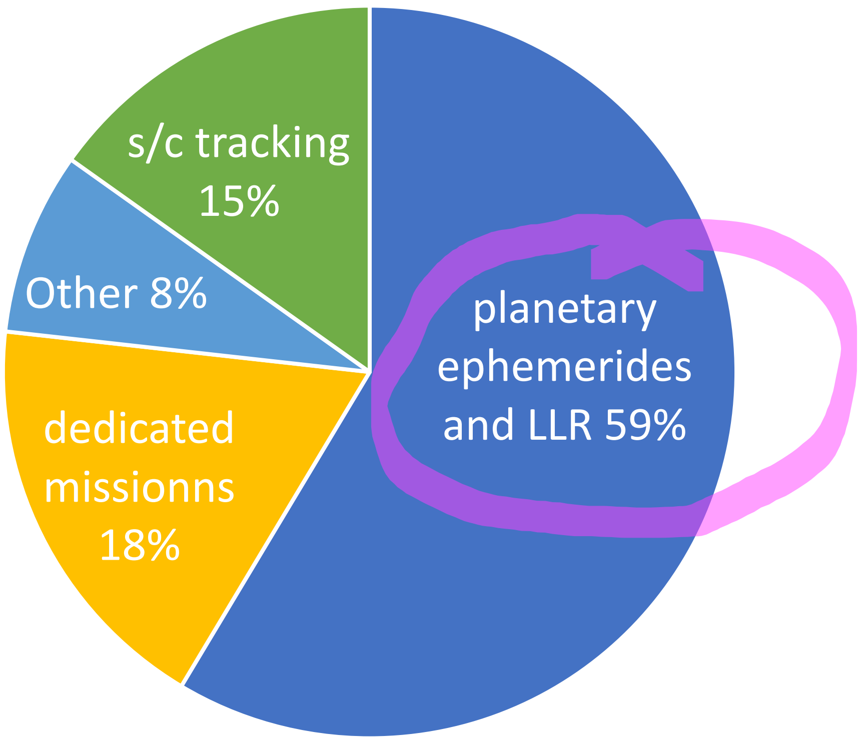
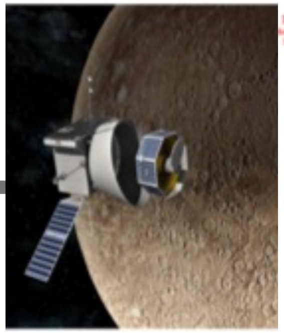
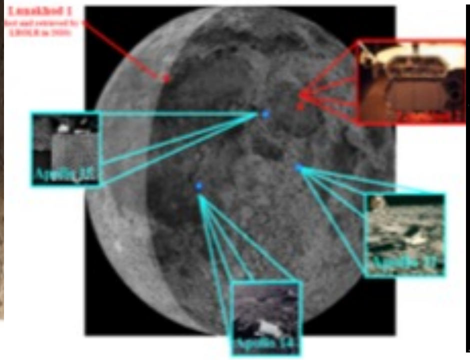
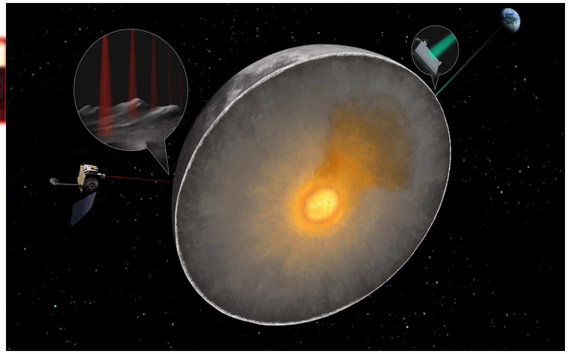
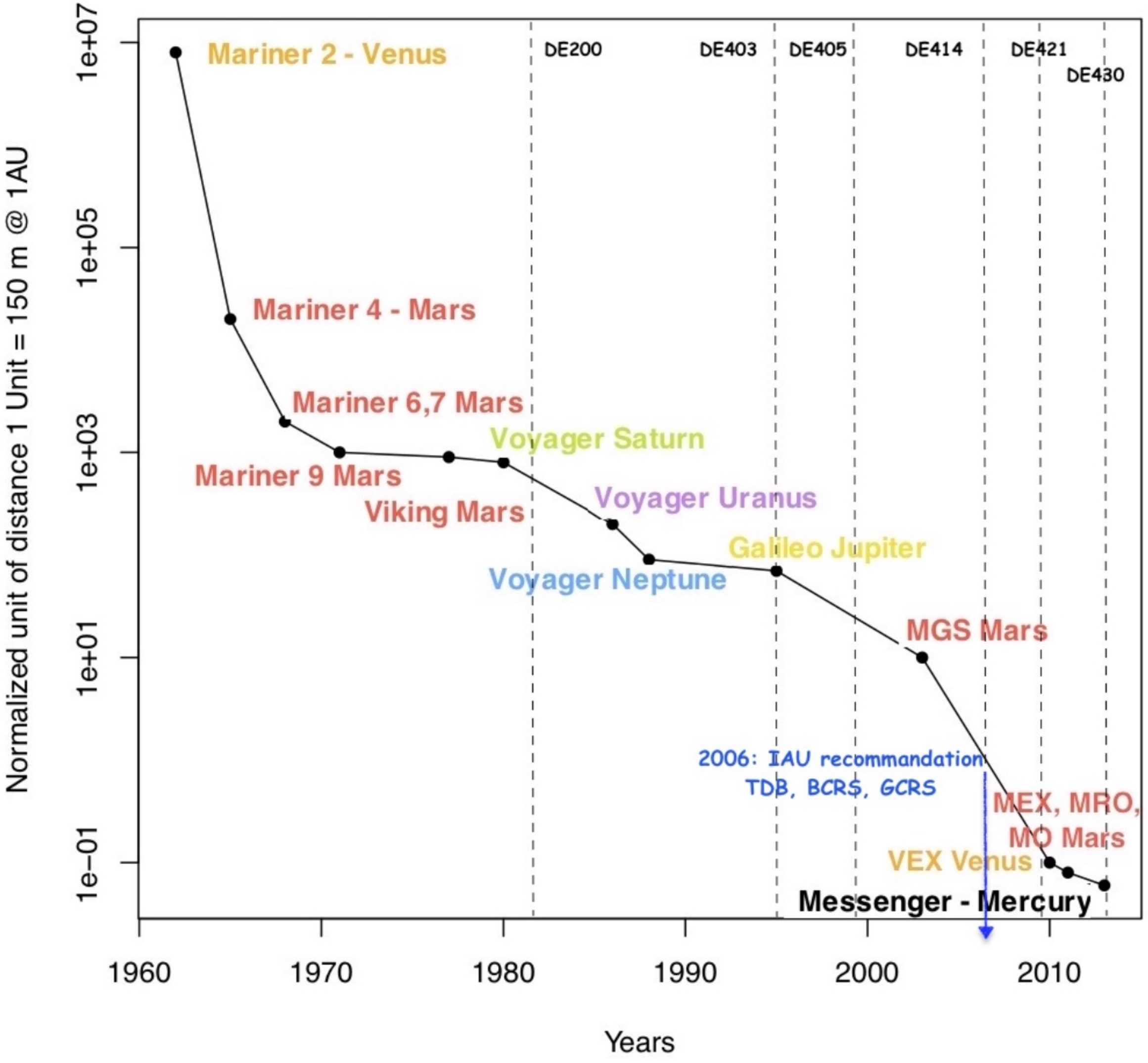


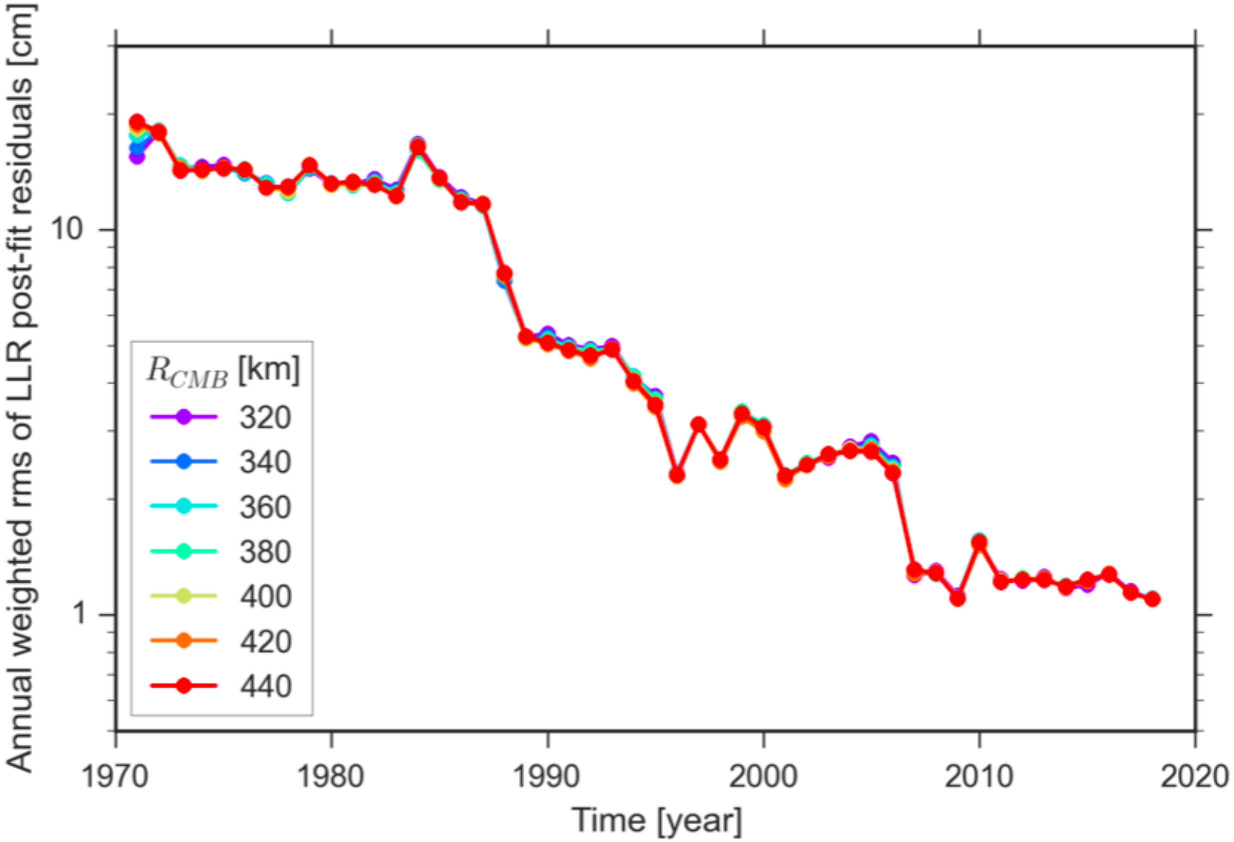
Solar system is a good place for testing gravity in weak field !!









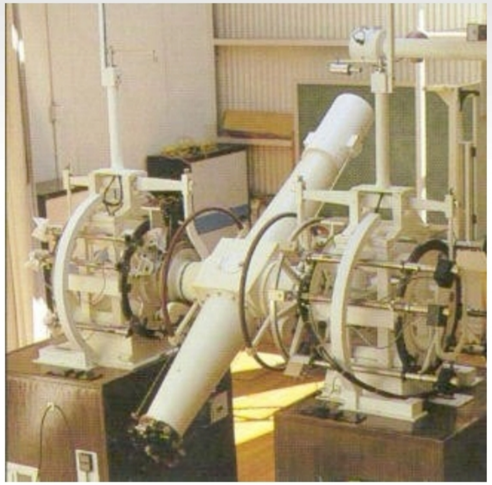


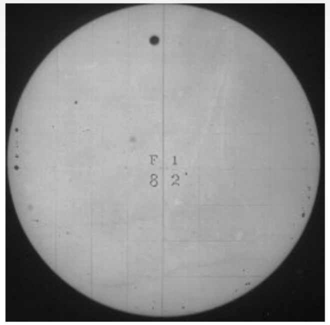
(Viswanathan et al. 2019)

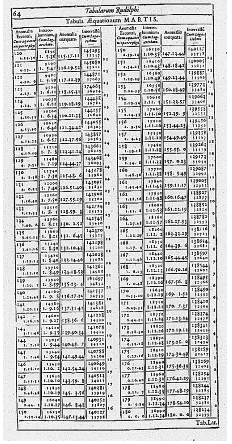
Planetary dynamics in the past

Until the 60's , good measurements of angles and orbital periods

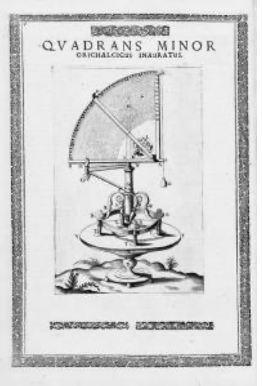
- Sun Transit
- Optical Astrometry
- Transit circle, photos
- Good time resolution







Kepler's Rudoplhin tables



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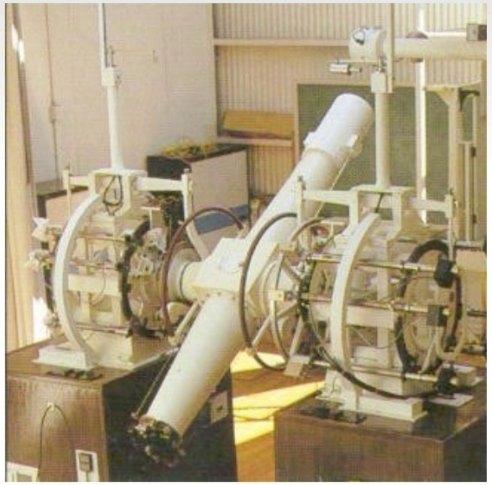

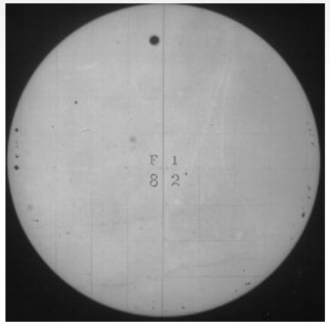

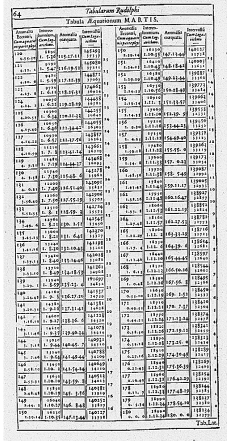

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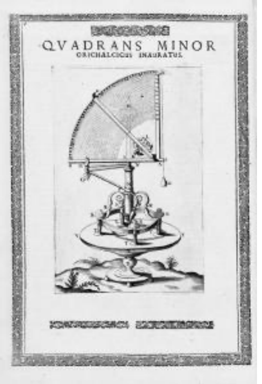
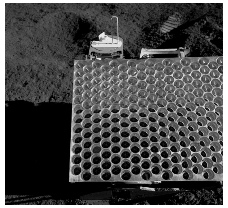
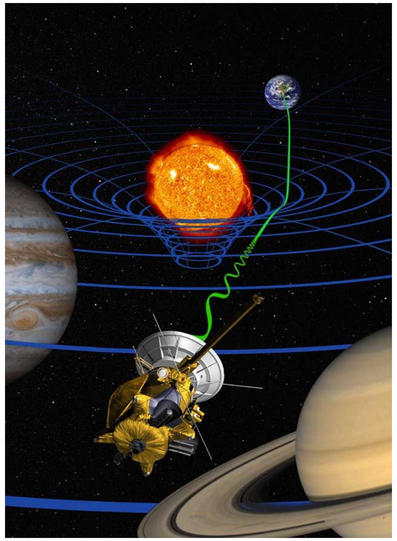


Space exploration

with direct measurements of the distances





- 1958, First antenna for Pionner 8 and 9

- 1960, First radar tracking on planetary surfaces (Mars, Venus)

3 generations of ephemerides

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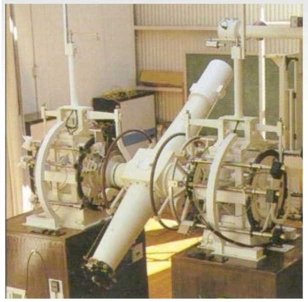

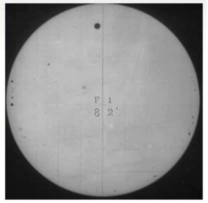

Space exploration

Direct measurements of the distances

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Ephemerides	Gaillot Gaillot and Le Verrier (1913)		DE102 Standish (1983)		DE440/INPOP19a Park et al. (2021) Fienga et al. (2019a)	
Data span	1800-1913		1913-1983		1924-2021	
	angle	distance Earth-	angle	distance Earth-	angle	distance Earth-
	"	km	"	km	"	km
Mercury	1	450	0.050	5	0.002	0.004
Venus	0.5	100	0.050	2	0.002	0.006
Mars	0.5	150	0.050	0.050	0.001	0.0015
Jupiter	0.5	1400	0.1	10	0.010	0.020
Saturn	0.5	3000	0.1	600	0.001	0.020
Uranus	1	12700	0.2	2540	0.050	10
Neptune	1	22000	0.2	4400	0.050	50
Pluto	1	24000	0.2	4800	0.050	2400

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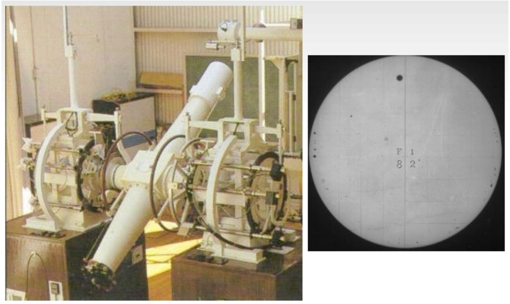

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Data span	1800-1913		1913-1983		1924-2021		
	angle	distance Earth-	angle	distance Earth-	angle	distance Earth-	$\Delta\dot{\varpi}$
	”	km	”	km	”	km	”.yr^{-1}
Mercury	1	450	0.050	5	0.002	0.004	0.43
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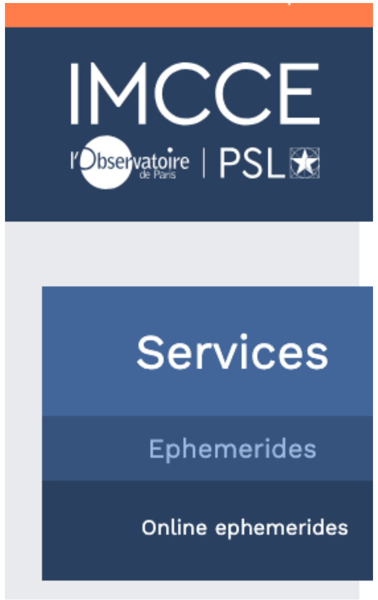
Saturn	0.5	3000	0.1	600	0.001	0.020	0.010
Uranus	1	12700	0.2	2540	0.050	10	0.005
Neptune	1	22000	0.2	4400	0.050	50	0.0033
Pluto	1	24000	0.2	4800	0.050	2400	0.0027

Mercury advance of perihelia

Modern ephemerides

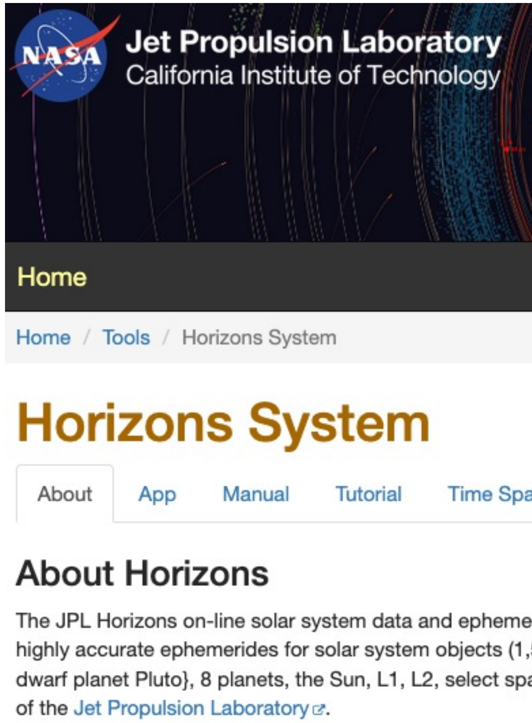


What are PLE = Planetary and Lunar Ephemerides ?

- Tables of positions and velocities of bodies. → Dynamical models adjusted to observations
- Solar system bodies : Main planets, asteroids, Kuiper Belt objects, natural satellites
- Euler angles of Moon rotation

What for ?

- Celestial mechanics and reference frames Tests of fundamental physics
- Planetology: physics of asteroids, Moon Solar physics
- Preparation and navigation of space missions
- Impact for example in SGWB detection with pulsar timing
- Paleoclimatology and geological time scales
- Other topics: preparation of stellar occultations, public outreach

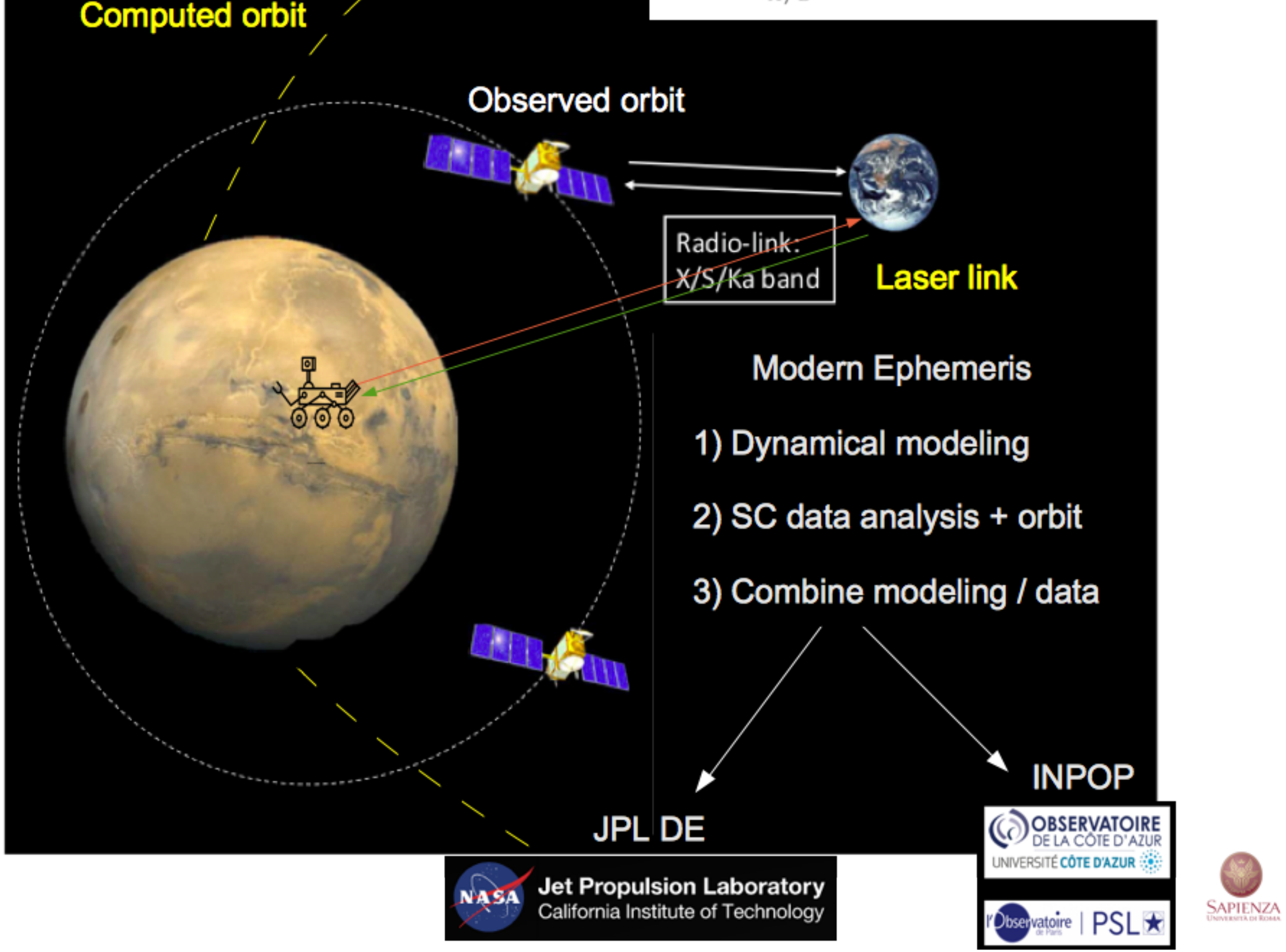


Modern ephemerides

3 independent reference ephemerides:

- JPL DE(*)
  https://ssd.jpl.nasa.gov/
- OP/OCA INPOP(*)
  https://www.imcce.fr/inpop
- Russian EPM
  https://iaaras.ru/en/dept/ephemeris/epm

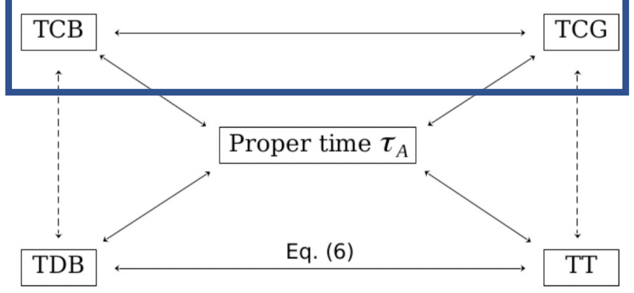
$$\ddot{x}_{Planet} = \sum_{A\neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta,\gamma,c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}}$$



(*) Full or partial reprocess of s/c data

PLE GR framework

- IAU 2000 recommandations (Soffel et al. 2003)
- Kinematically fixed BCRS (TCB)
- Harmonic gauge conditions
- SSB definition solved at J2000
- Time-scales TCB, TCG → TDB, TT



$$
\begin{aligned}
g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}) \\
g_{0i} &= -\frac{4w_i}{c^3} + \mathcal{O}(c^{-4}) \\
g_{ij} &= \delta_{ij}\left(1 + \frac{2w}{c^2}\right) + \mathcal{O}(c^{-3}),
\end{aligned}
$$

Space-time metric g in weak-field approximation with harmonic gauge (see part II)

(Einstein-Infeld-Hoffmann-Droste-Lorentz equation)

μ_A = G x gravitational mass, m_g

$$
\begin{aligned}
\boldsymbol{a}_T = &- \sum_{A\neq T} \frac{\mu_A}{r_{AT}^3}\boldsymbol{r}_{AT} \\
&- \sum_{A\neq T} \frac{\mu_A}{r_{AT}^3 c^2}\boldsymbol{r}_{AT}\Bigg\{ v_T^2 + 2v_A^2 - 4\boldsymbol{v}_A.\boldsymbol{v}_T - \frac{3}{2}\left(\frac{\boldsymbol{r}_{AT}.\boldsymbol{v}_A}{r_{AT}}\right)^2 \\
&\qquad - \frac{1}{2}\boldsymbol{r}_{AT}.\boldsymbol{a}_A - 4\sum_{B\neq T}\frac{\mu_B}{r_{TB}} - (\sum_{B\neq A}\frac{\mu_B}{r_{AB}}\Bigg\} \\
&+ \sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}^3}\left[4\boldsymbol{r}_{AT}.\boldsymbol{v}_T - 3\boldsymbol{r}_{AT}.\boldsymbol{v}_A\right](\boldsymbol{v}_T - \boldsymbol{v}_A) \\
&+ \frac{7}{2}\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}}\boldsymbol{a}_A,
\end{aligned}
$$

SSB definition

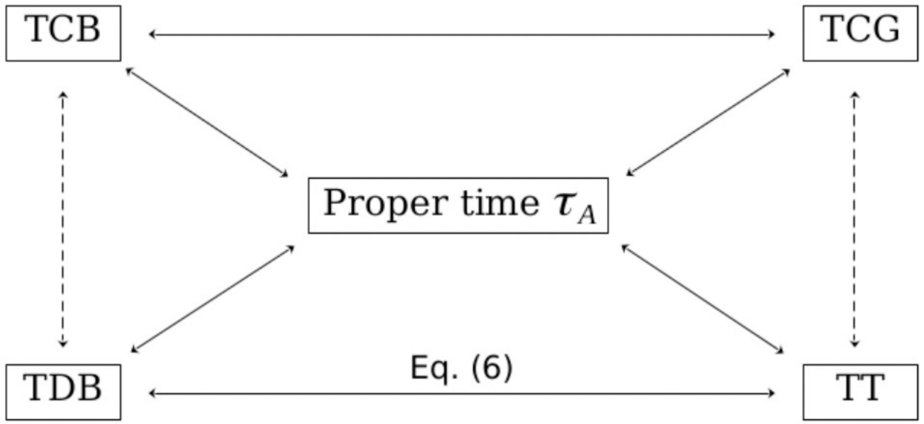
μ_i = G x Inertial mass, m_i

$$
\begin{cases}
\sum_i \mu_i^* \boldsymbol{r}_i = 0 \\
\sum_i \mu_i^* \boldsymbol{v}_i + \dot{\mu}_i^* \boldsymbol{r}_i = 0
\end{cases}
$$

$$
\begin{cases}
\mu_i^* = \mu_i\left(1 + \frac{v_i^2}{2c^2} - \frac{1}{2c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\right) \\
\dot{\mu}_i^* = \frac{\mu_i}{2c^2}\left(\sum_{j\neq i}\mu_j \frac{(\boldsymbol{r}_j - \boldsymbol{r}_i).(\boldsymbol{v}_j + \boldsymbol{v}_i)}{r_{ij}^3}\right)
\end{cases}
$$

GR framework for planetary and lunar ephemerides (details slide 8)

- Time-scales TCB, TCG → TDB, TT
- Equation of motion for point mass bodies
- Equation of motion for photon
- SSB definition



$$\frac{d(TT-TDB)}{d(TDB)} = (L_B + \frac{1}{c_2}a)(1 + L_B - L_G) - L_G + \frac{1}{c_4}b,$$

$$a = -\frac{1}{2}v_T^2 - \sum_{A \neq T} \frac{\mu_A}{r_{AT}}$$

$$b = -\frac{1}{8}v_T^4 + \frac{1}{2}\left[\sum_{A \neq T} \frac{\mu_A}{r_{AT}}\right]^2 + \sum_{A \neq T} \frac{\mu_A}{r_{AT}} \left\{ 4\boldsymbol{v}_T.\boldsymbol{v}_A - \frac{3}{2}v_T^2 - 2v_A^2 + \frac{1}{2}\boldsymbol{a}_A.\boldsymbol{r}_{AT} + \frac{1}{2}\left(\frac{\boldsymbol{v}_A.\boldsymbol{r}_{AT}}{r_{AT}}\right)^2 + \sum_{B \neq A} \frac{\mu_B}{r_{BA}} \right\},$$

There are two concepts of masses

1. The inertial mass m_i such as $\mathbf{F} = m_i \ \mathbf{a}$ ← This relation is independent from the type of force

m_i is the mass of the particule at rest. It is the quantity of mass (or energy) of the particule independently from its motion. It is identical for all the inertial frames.

2. The gravitationnal mass m_g such as $\mathbf{F}_g = m_g \ \mathbf{g}$ ← This relation is specific to the gravitationnal force

m_g can be seen as the gravitationnal charge of the particule

(more in part II)

GR framework for PLE: light path

- Equation of motion for photon
- Frame-dragging effect (Lense-Thirring)

μ_A = G x Inertial mass, m_A

$$c(t_r - t_e)_{GRT} = R + \sum_A 2\frac{\mu_A}{c^2} \ln \frac{\boldsymbol{n}\cdot\boldsymbol{r}_{rA} + r_{rA} + \frac{4\mu_A}{c^2}}{\boldsymbol{n}\cdot\boldsymbol{r}_{eA} + r_{eA} + \frac{4\mu_A}{c^2}},$$

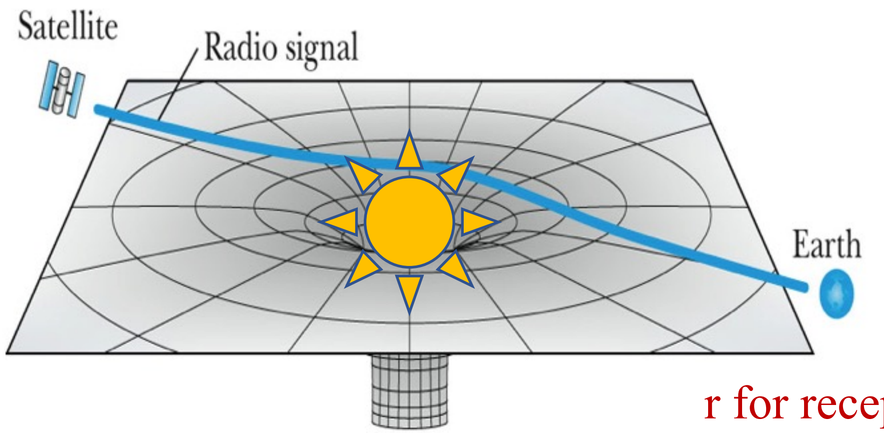
$$\boldsymbol{n} = (\boldsymbol{r}_r - \boldsymbol{r}_e)/\|\boldsymbol{r}_r - \boldsymbol{r}_e\|,$$

$$\boldsymbol{r}_{iA} = \boldsymbol{r}_i - \boldsymbol{r}_A \text{ and } r_{iA} = \|\boldsymbol{r}_{iA}\| \text{ with } i = e \text{ or } r.$$

$$\boldsymbol{a}_{LT} = \frac{2GS}{c^2 r^3}\left[\frac{3\boldsymbol{k}.\boldsymbol{r}}{r^2}(\boldsymbol{r}\wedge\boldsymbol{v}) - (\boldsymbol{k}\wedge\boldsymbol{v})\right]$$

S, Sun angular momentum and ***S*** = *S* ***k*** with ***k*, ** Sun pole direction
r,v position and velocity of the planet relative to the Sun

e for emission, r_e



r for reception, r_r

(see part II for derivation)

- planets

How do we built planetary and lunar ephemerides ?

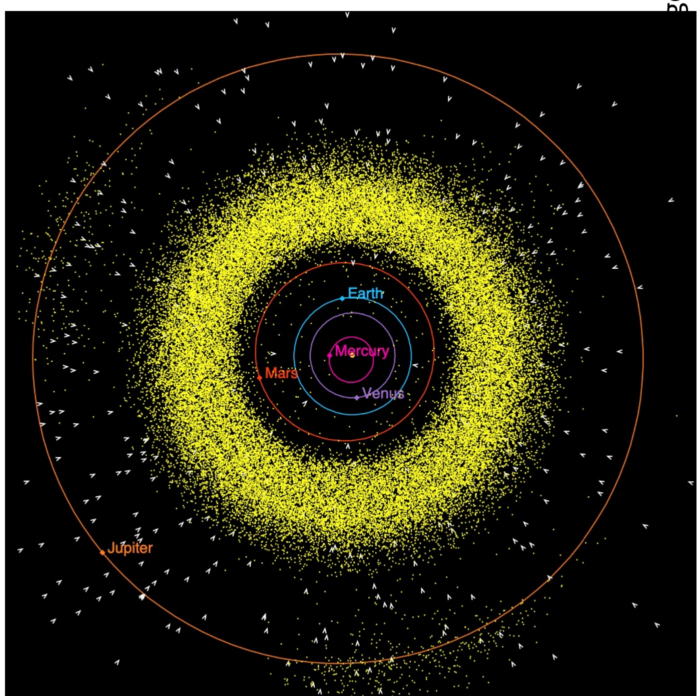
- Numerical integration of the (Einstein-Imfeld-Hoffmann, c^{-4} PPN approximation) equations of motion.

$$\ddot{x}_{Planet} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}}$$

- Adams-Cowell in extended precision
- 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, $J_2^{\odot}$, Earth rotation (Euler angles, specific INPOP)
- Moon: orbit and librations
- Simultaneous numerical integration TT-TDB, TCG-TCB
- Testing GR and alternative theories, asteroid masses, solar physics
- Fit to observations in ICRF over 1 cy (1914-2014) including LLR
- IERS conventions

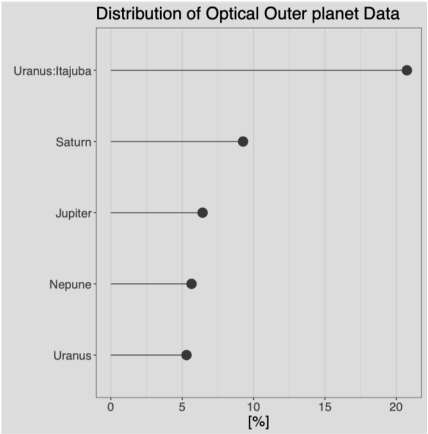
- From 400 to 800 estimated parameters per solution

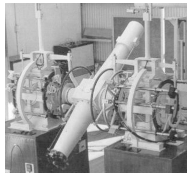
gov/diagrams/



Datasets and PE accuracies

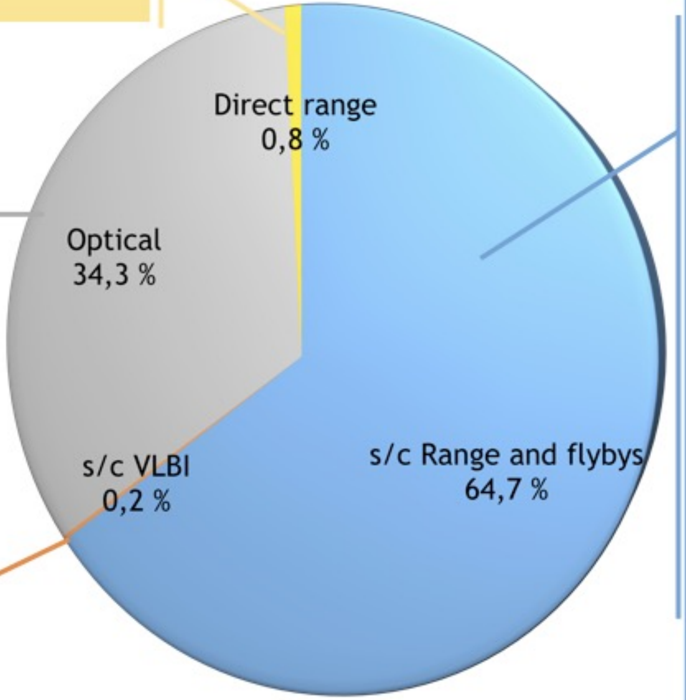
- 2 types : s/c tracking (distances, angles) and optical astrometry (angles)
- Inner planets : s/c tracking
- Outer planets: optical astrometry (U,N) + tracking (J,S)

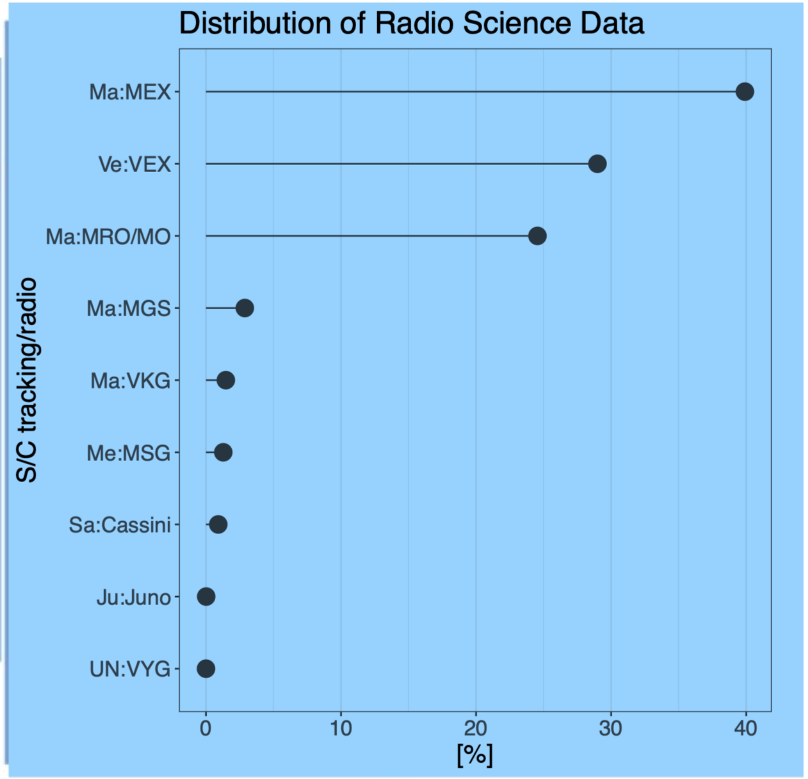


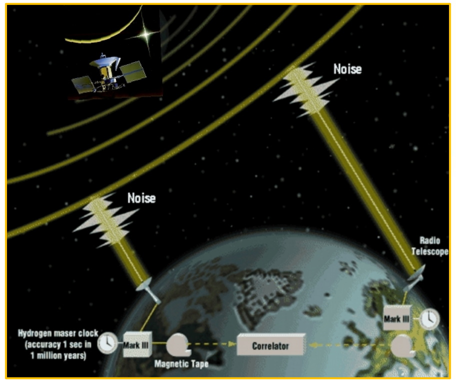


- 1965-1997
- Mercury,Venus
- *1 km*

- 1924-2014
- Jupiter, Saturn, Uranus, Neptune, Pluto
- planet and satellites
- 300 mas



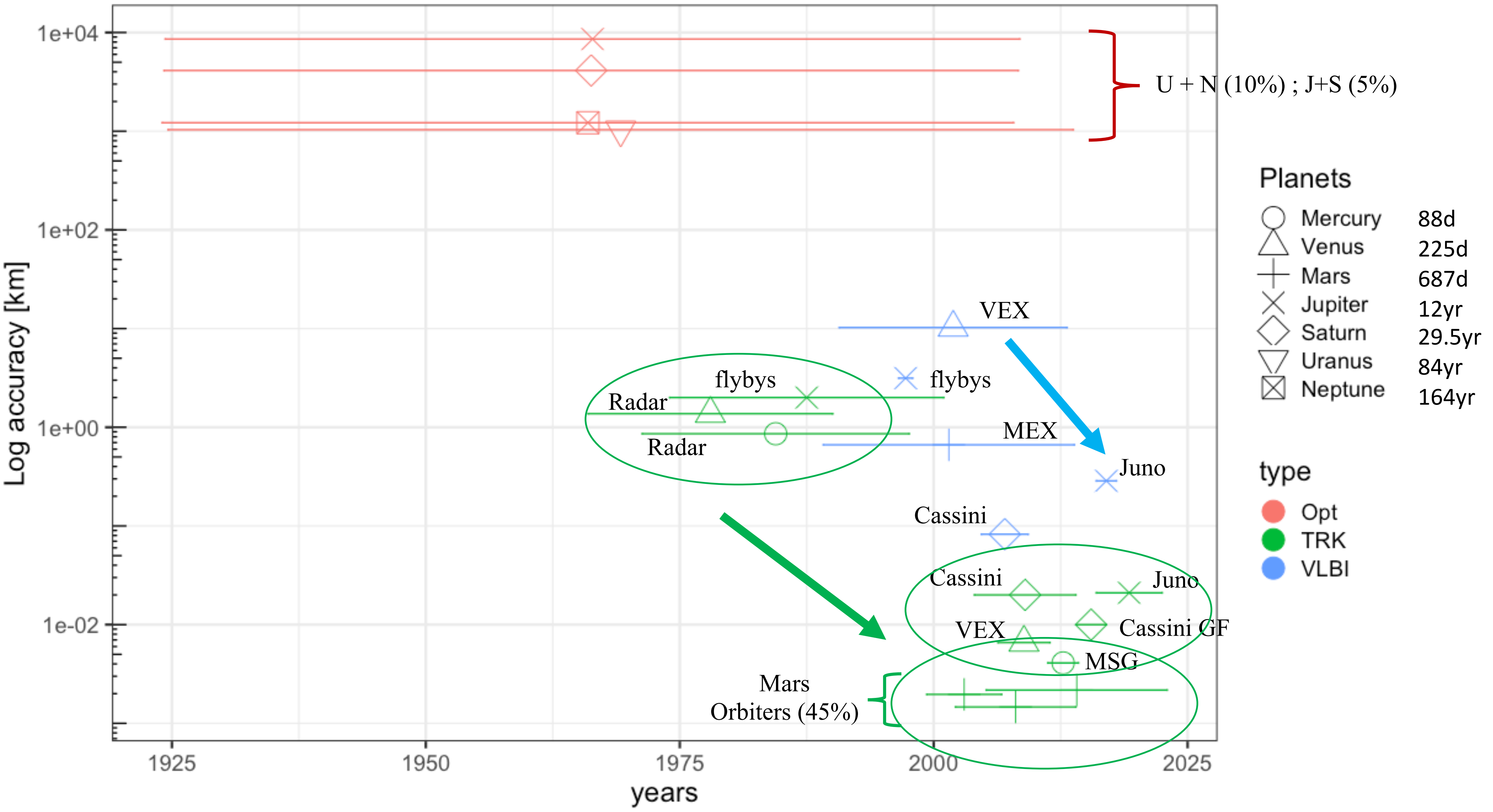




- 1990-2010
- Mars, Venus, Jupiter, Saturn
- 1 mas to 10 mas

44.4% of the full PE datasets are Mars s/c tracking observations (09/09/2026)

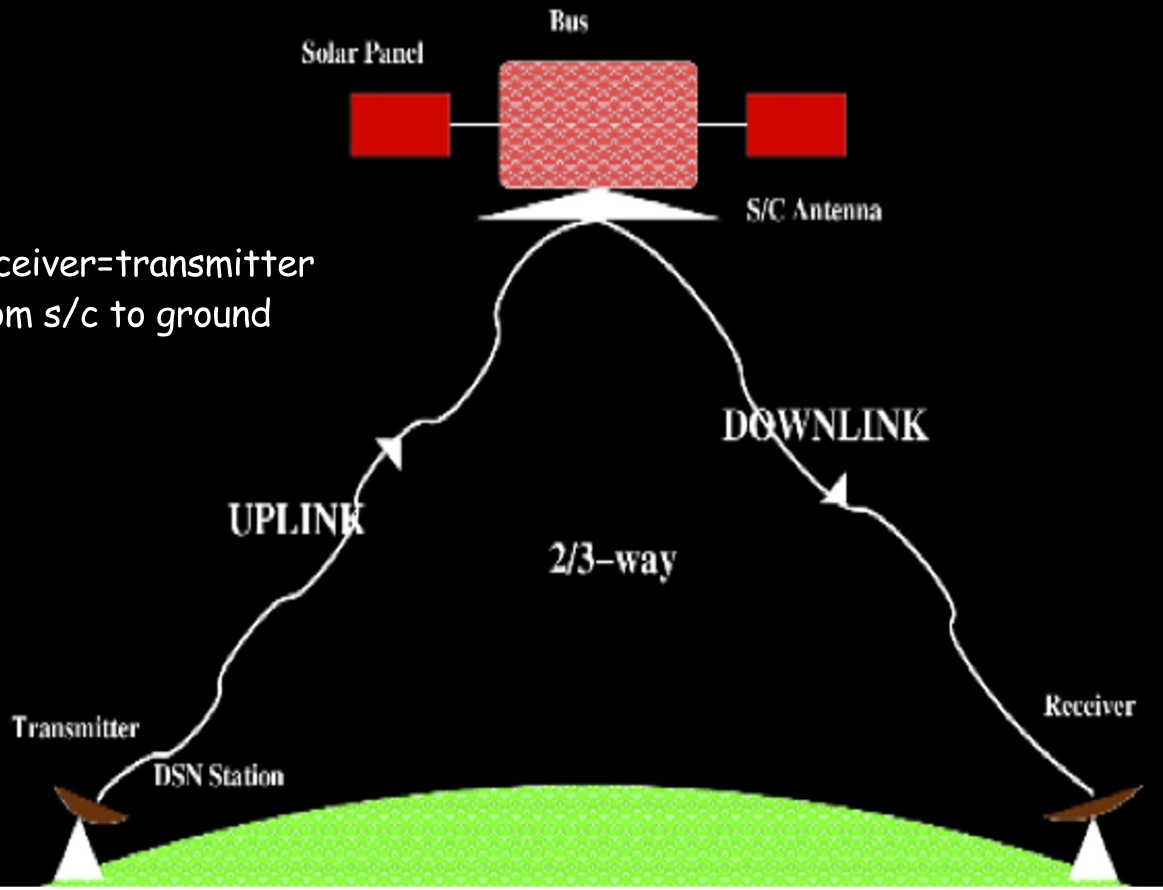
Datasets and PE accuracies



s/c navigation or radio science

- 1-,2-,3-way Doppler:
  - → received signal differ from transmitted signal
  - → doppler shift
- 2-way range:
  - → signal travel time
  - → range measurements
- Experiments: gravity field, atmosphere, ephemerides, fundamental physics, etc..

2-way : receiver=transmitter
1-way : from s/c to ground



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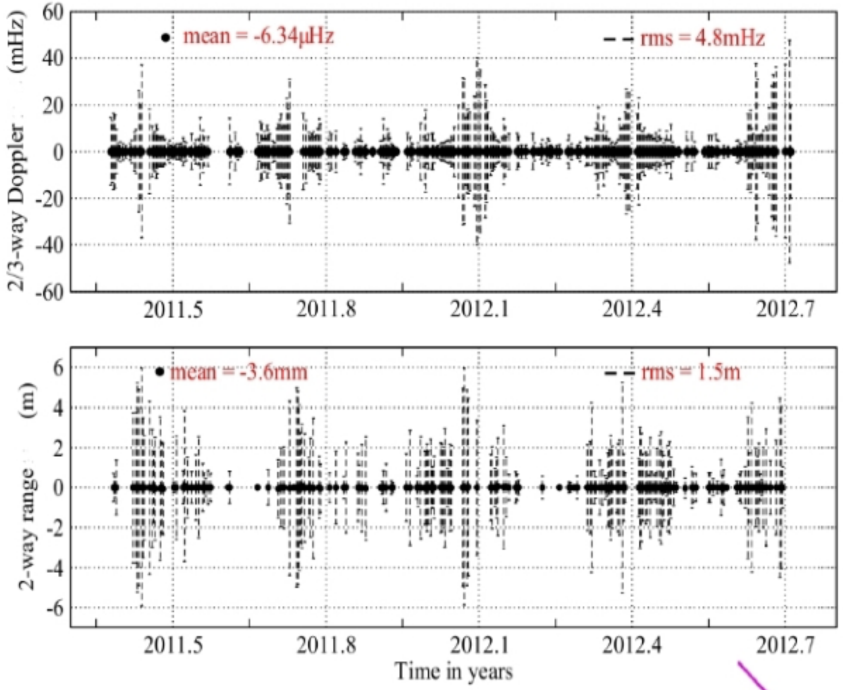


2-way Doppler

→ Differences between estimated **velocities** of s/c orbiting the planet and the observed Doppler shift

2-way range

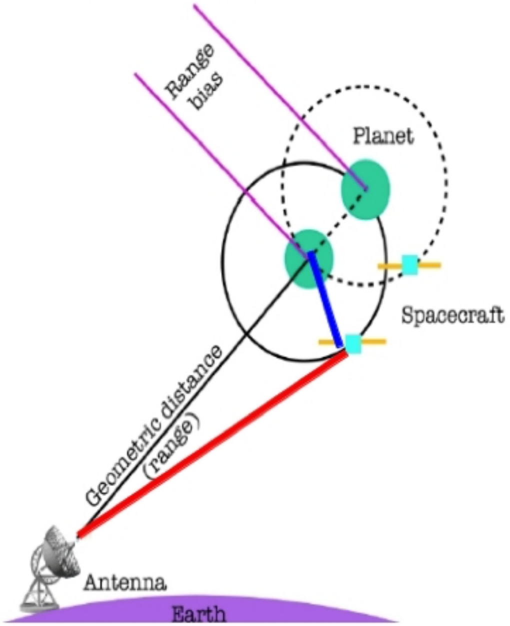
→ Differences between estimated **distances of s/c** **orbiting the planet** and the Earth observed time delay

↓

Orbit of the s/c about the planet very well known

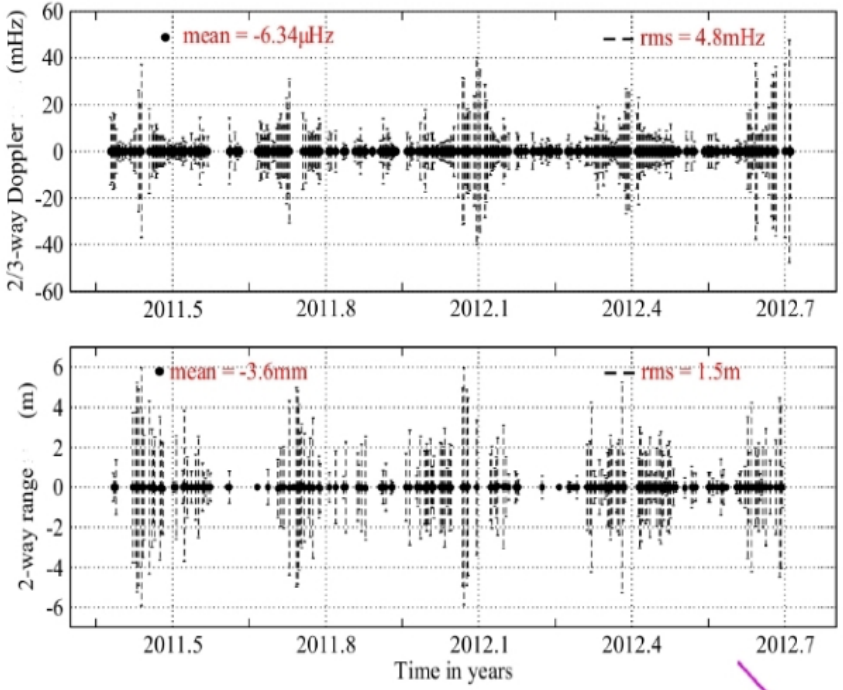
↓

Constraints on **distances between the planet and the Earth**



s/c navigation or radio science

Modeling:
Solar plasma
GR
Troposphere
Earth rotation
Atmopheric drag
Manouvers, SRP



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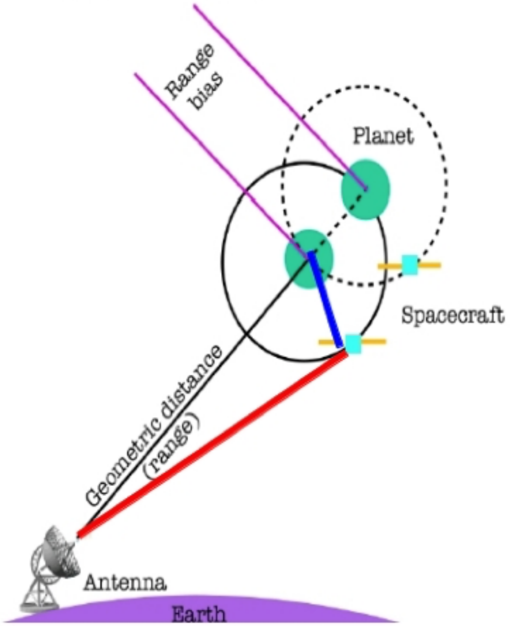
Differences between estimated **distances of s/c** **orbiting the planet** and the Earth observed time delay

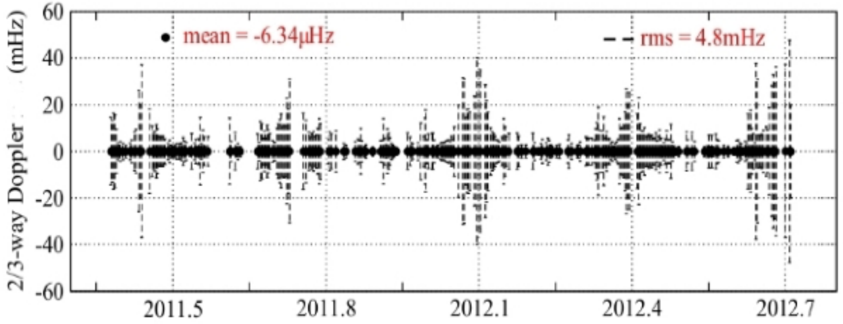
Orbit of the s/c about the planet very well known

Constraints on **distances between the planet and the Earth**



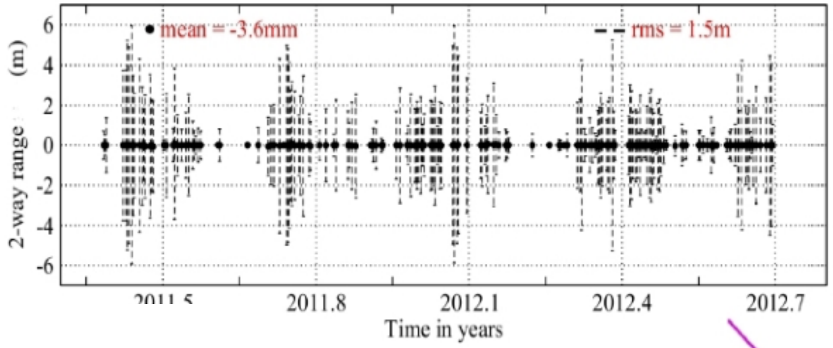
s/c navigation or radio science

Modeling:
Solar plasma
GR
Troposphere
Earth rotation
Atmopheric drag
Manouvers, SRP



2-way Doppler

→ Differences between estimated velocities of s/c orbiting the planet and the observed Doppler shift



2-way range

→ Differences between estimated distances of s/c **orbiting the planet** and the Earth observed time delay

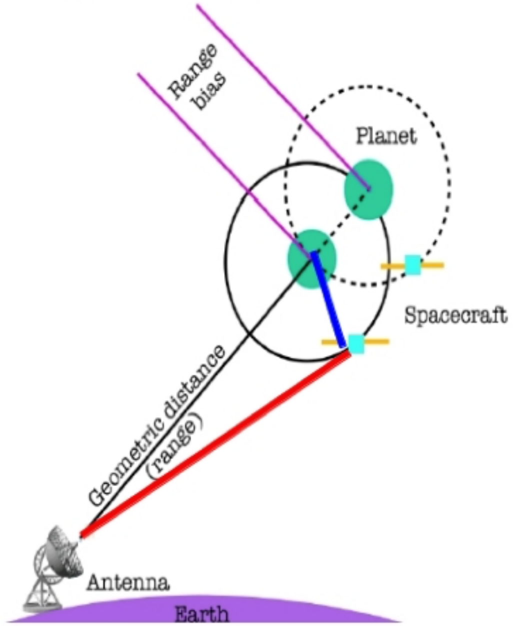
↓

Orbit of the s/c about the planet very well known

↓

Constraints on distances between the planet and the Earth

Me	MSG$^{c,s/c}$	11-13	300	10 m
Ve	VEX^{p}	06-12	25000	10 m
Ma	MEX^{p}	05-20	56,000	2 m
	MGS^{c}	99-05		5 m
	MRO-MO^{c}	06-14		1 m
Sat	Cassini^{c}	04-14	200	20 m
		14-17	350	$<$ 10 m
Jup	Juno^{c}	16-20	14	10 m

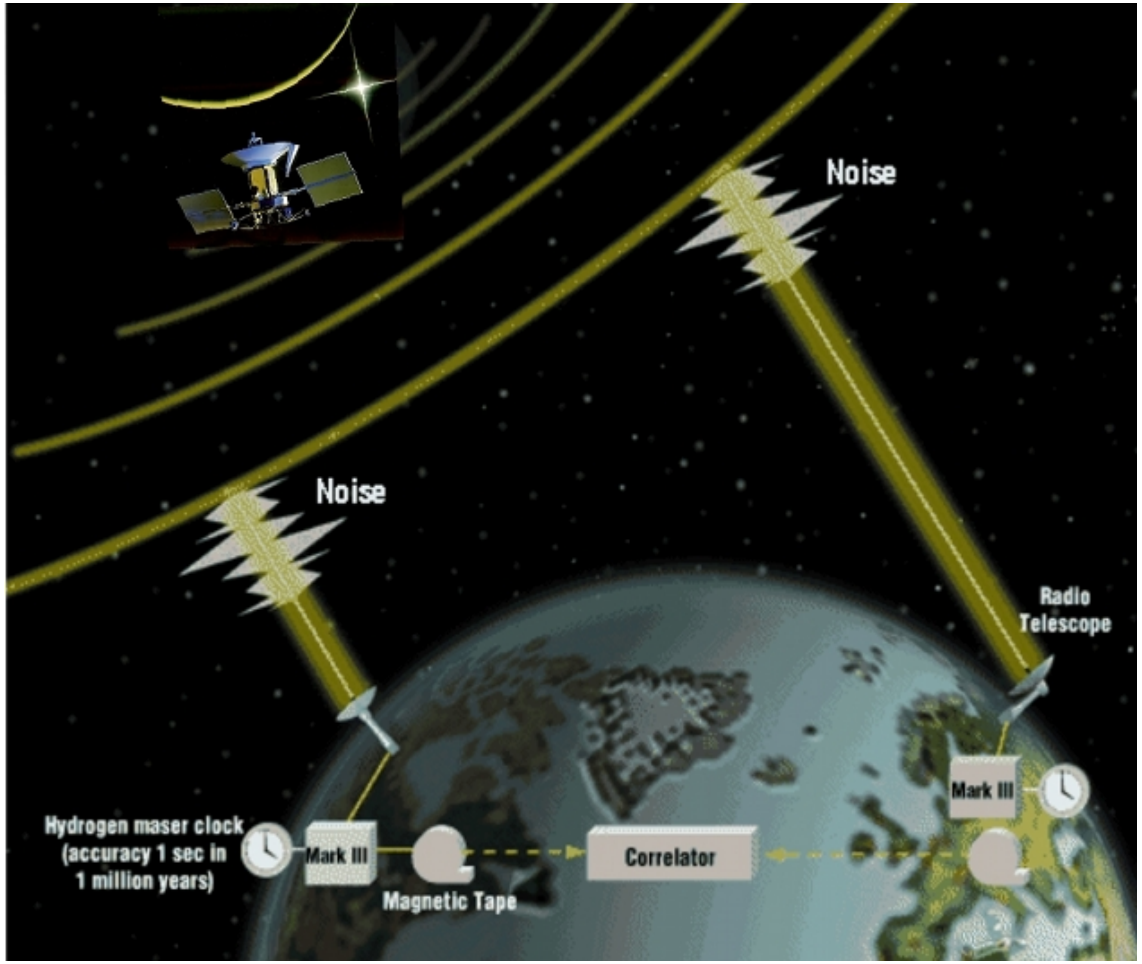


s/c navigation : Very Large Interferometry Baseline (VLBI)

- Very Long Base Interferometry 8GHz
- Relative positionning S/C versus ICRF QSO
- with S/C Orbit → angle Earth/planet/ICRF
- mas-level accuracy but very few points
- crucial for linking the planetary planes together

Ve	VEX	10-13	100	2 mas*
Ma	MEX-MGS-MO	00-15	200	2 mas
Jup	Galileo	96-97	12	11 mas
Sat	Cassini	04-14	30	1 mas

*: millisecond of arc

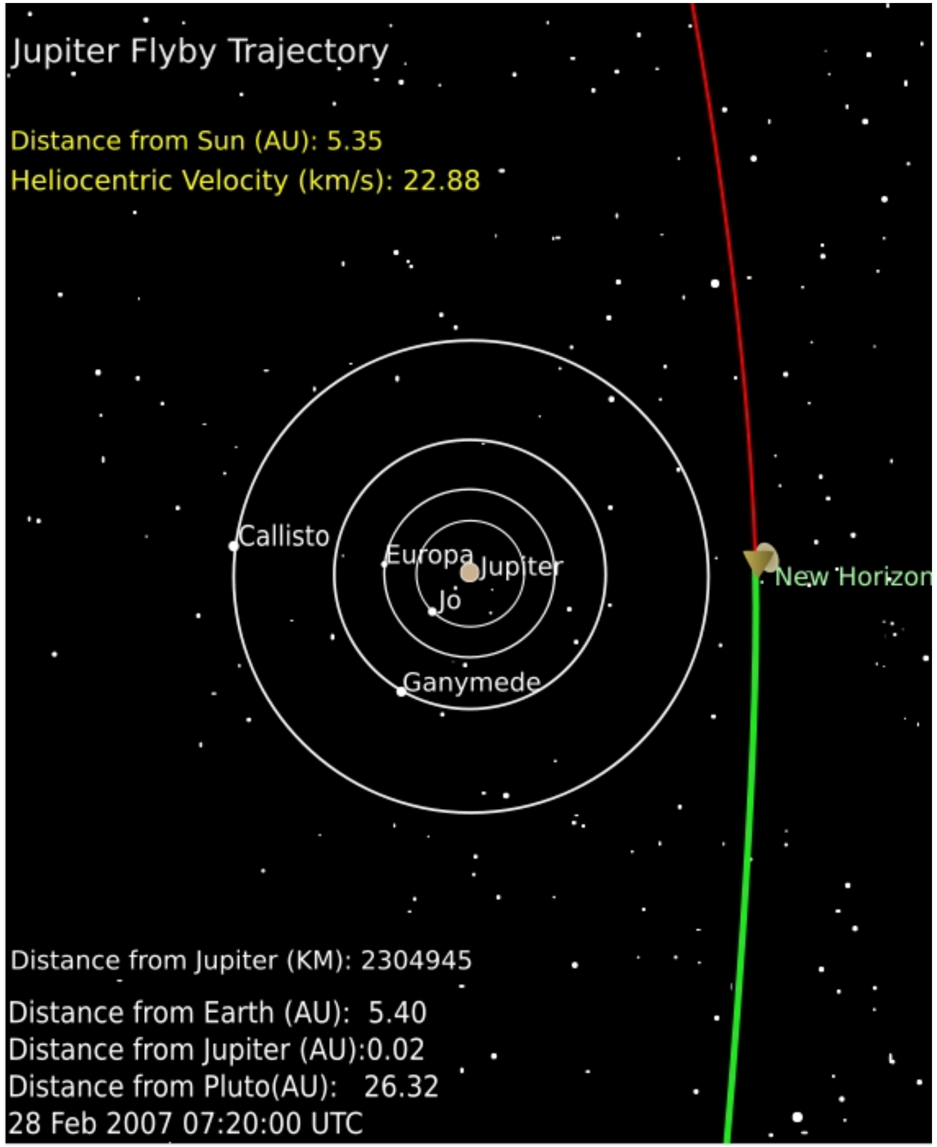


+ Juno VLBA

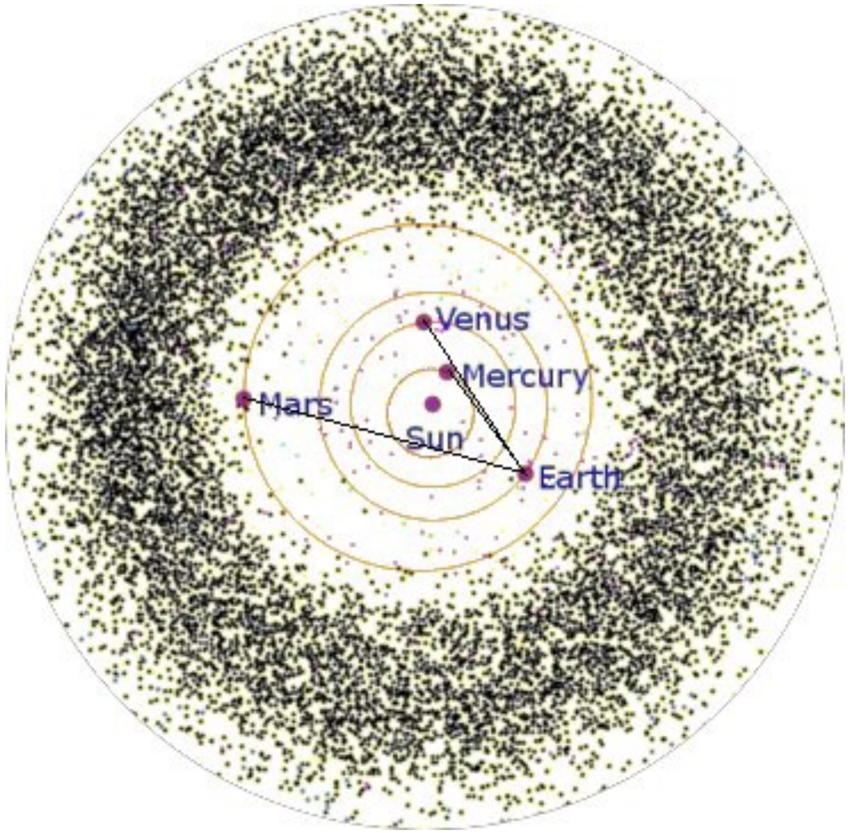
s/c navigation during flybys

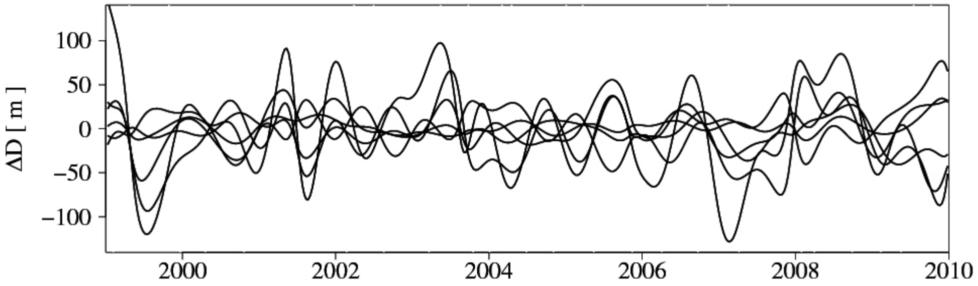
- Very short arc analysis
- Combination of Doppler, Range, VLBI
- with S/C Orbit → angle and distances Earth/planet/ICRF
- Very very few points
- crucial for outer planet orbits

	α, δ	ρ		
Mer	2 mas	1 m	MSG	3 (08-09)
Jup	5 mas	1.5 km	Pioneer, Cassini Voyager, Ulysses	5 (75-01)
Ura	10 mas	1000 km	Voyager	1 (86)
Nep	10 mas	2000 km	Voyager	1 (89)



Dynamical modeling beside GRT : Main belt





- More than 200,000 asteroids **with unknown masses** perturbed Mars orbit
- Mars orbit is the most accurately observed orbit thanks to numerous Mars missions since 2000
- meter-level accuracy measurements of the Earth-Mars distances since 2006

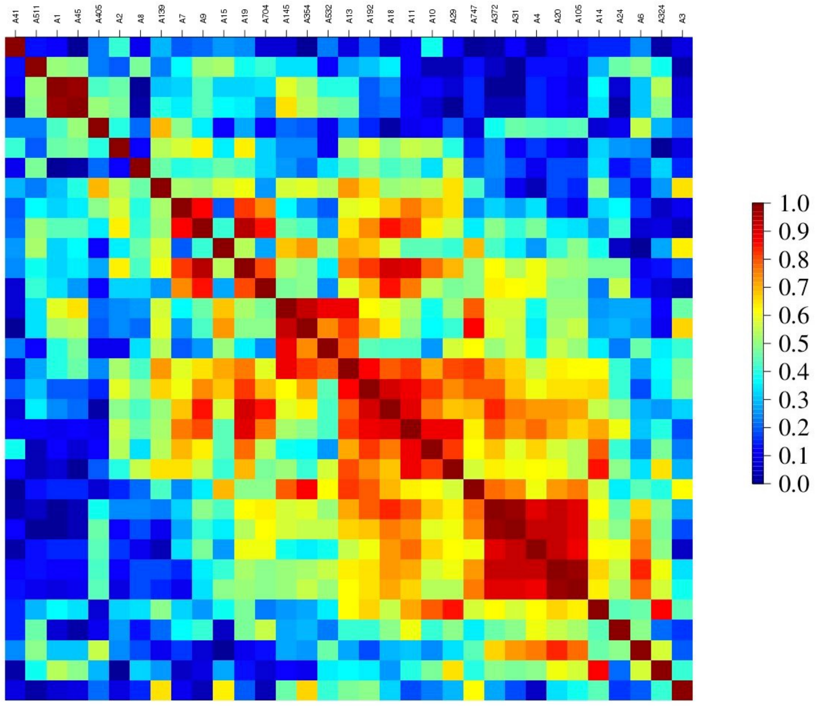
Missions	Agency	Period	# observations	One-way accuracy
Mars Express	European Space Agency	2005-2026	**40,109**	2 m
MGS	NASA	1999-2005	2,500	5 m
MRO/MO	NASA	2003-2014	18,000	1 m

Mars complex orbit

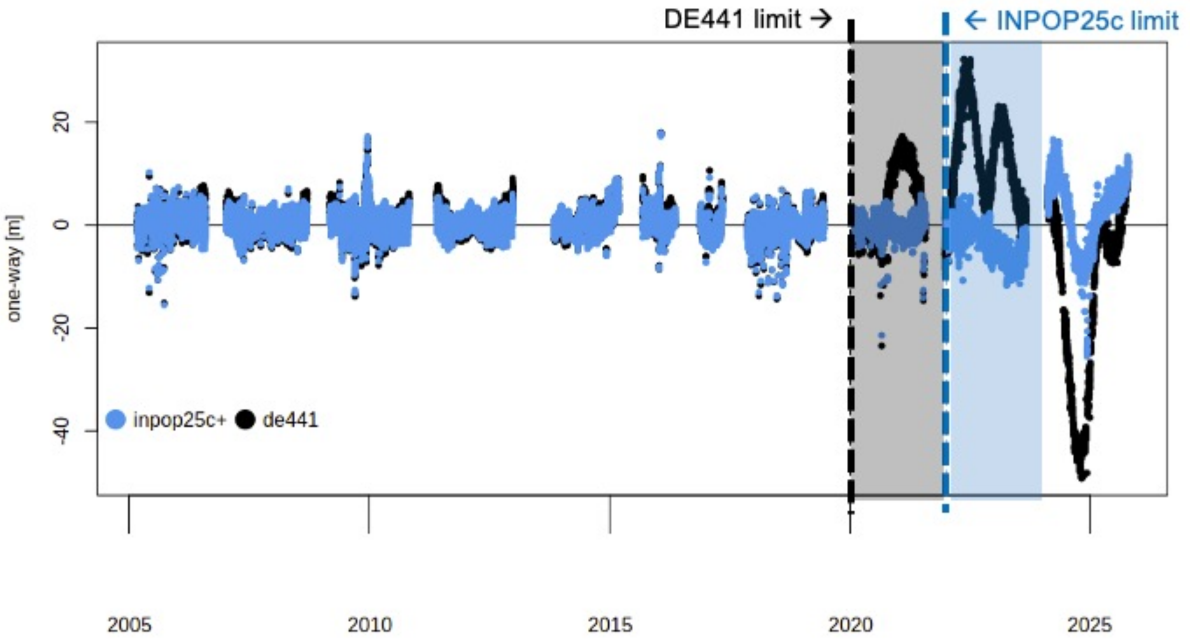
- Asteroid perturbations are included in the PLE EoM

- More than asteroid orbit, the asteroid masses are the big unknown !!
- Less than 100 asteroid masses are estimated with an uncertainties better than 33%

- Most of the PE overfit the asteroid masses → extrapolation disaster !

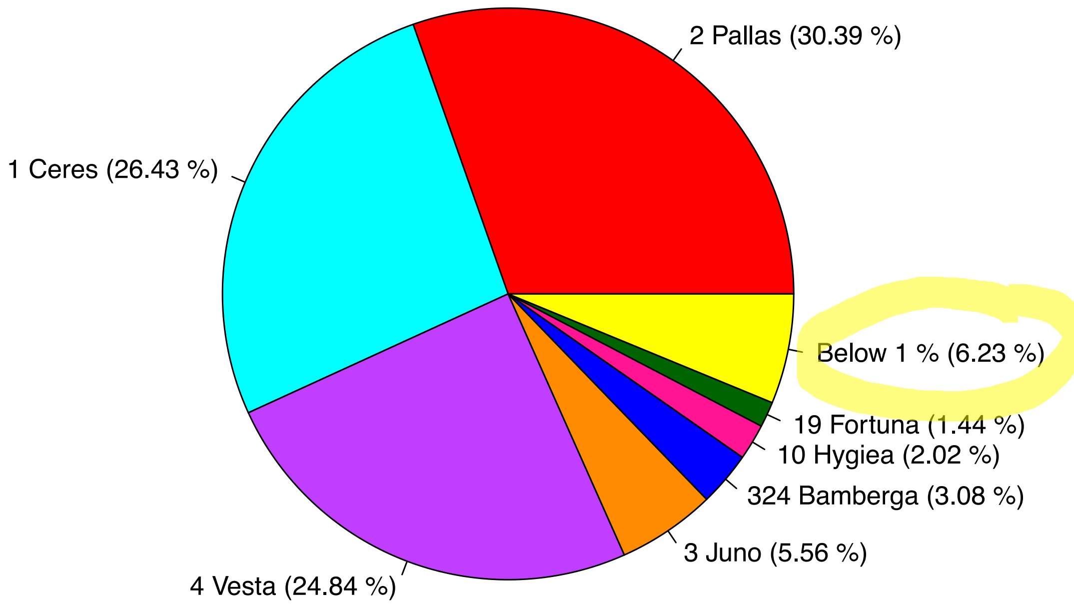


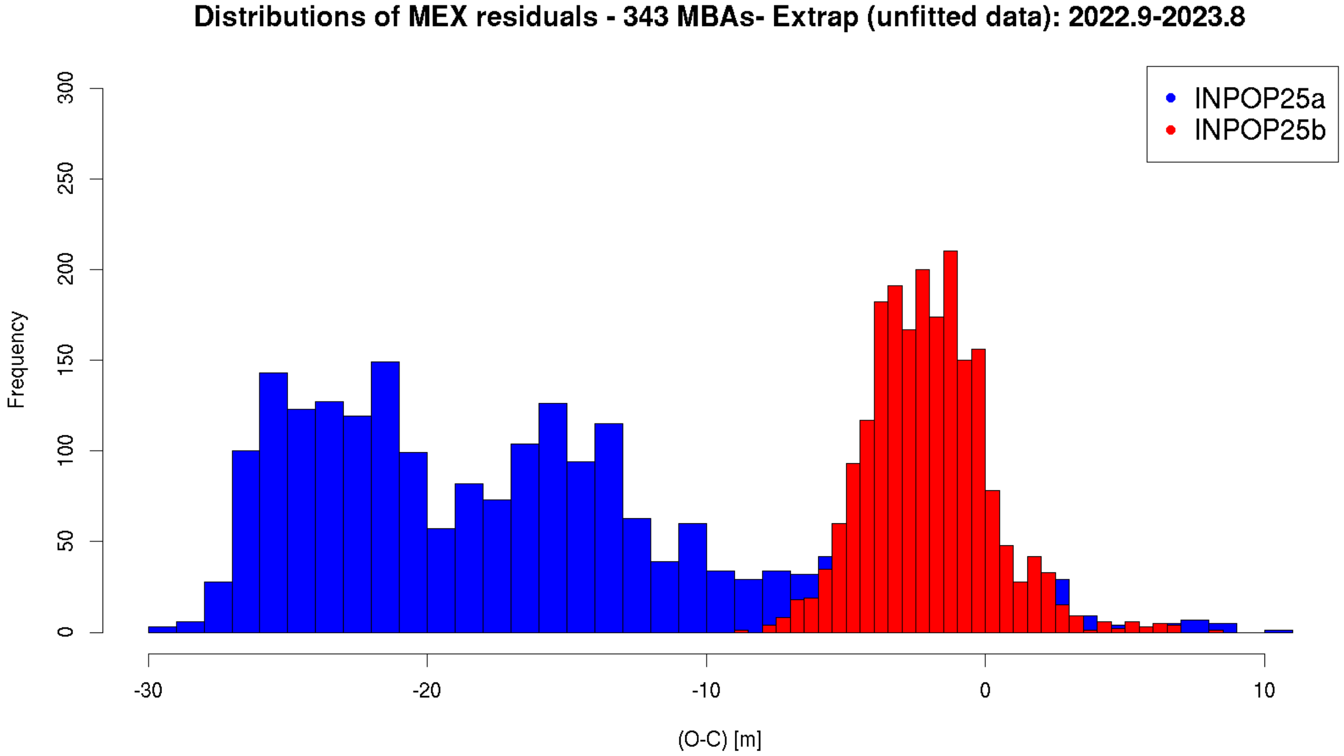
	INPOP17a 2018	INPOP19a 2020
Dynamics		
MBA	343 i.m. + 1 ring	343 i.m.
TNO	none	3 rings + 9 ind I21a: 509 ind
General Relativity	EIH	EIH + LT
Fit		
GM_A	153 BVLS + 1 ring	343 MC
TNO	none	1 ring mass I21a: 1 mass @ 509 ind.

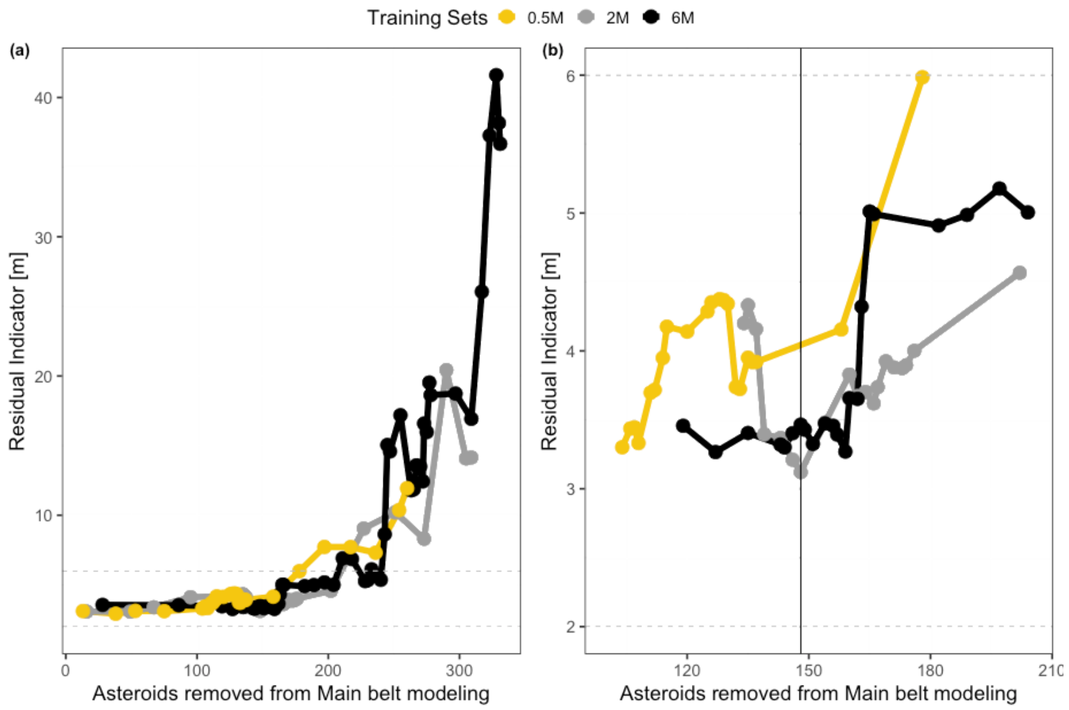


Mars complex orbit : Mariani et al 2025

- Use of Boosting decision trees for ranking by relative importance in the PLE asteroid modeling
- We reduce from 343 asteroids to 193

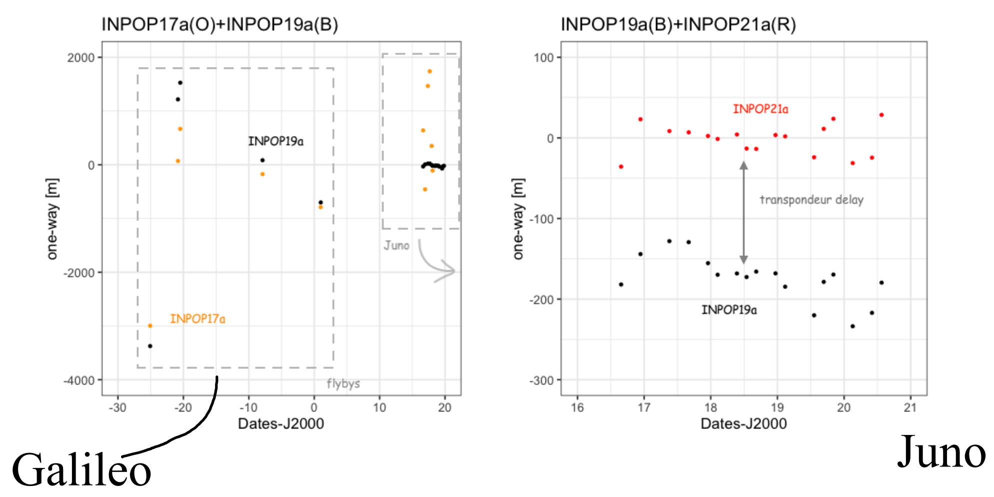






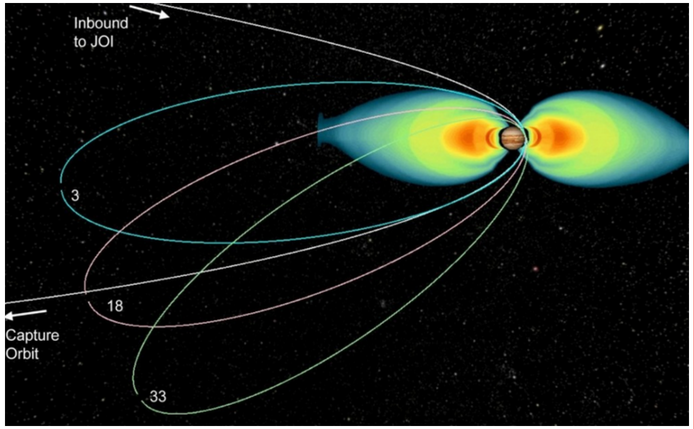
New outer solar system insights

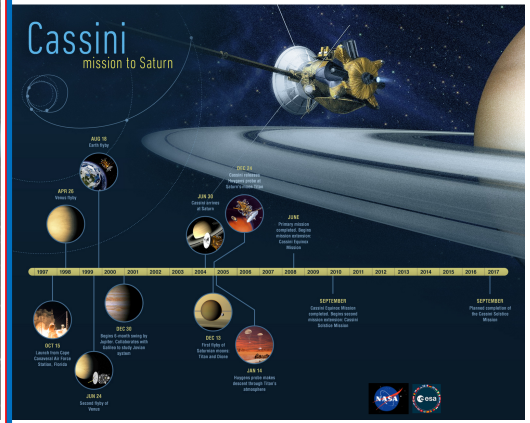
1 normal point per perijove



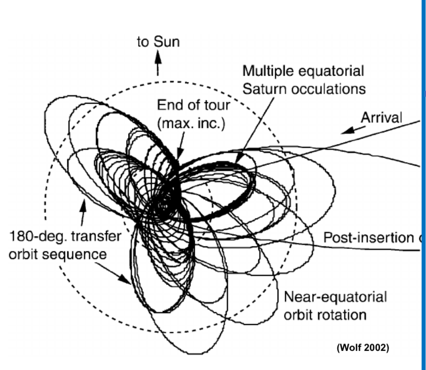
Important improvement of the Jupiter orbit:
from 2 kms to 20 m (± 150 m)

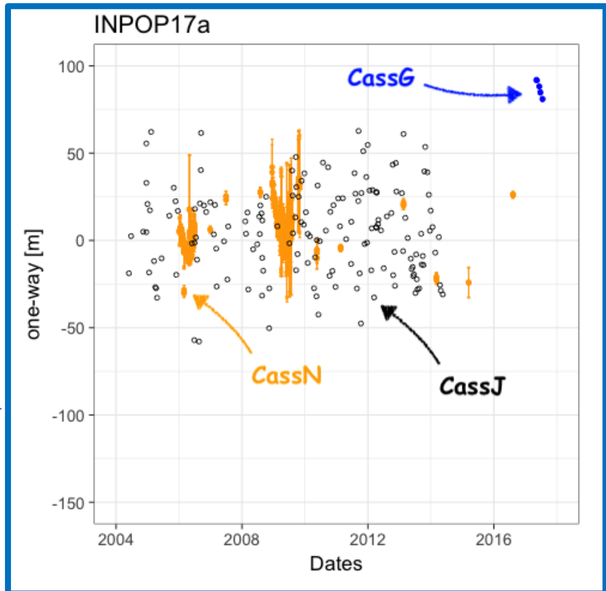
New view of the outer solar system: Juno (2016-) Cassini (2004-2017)





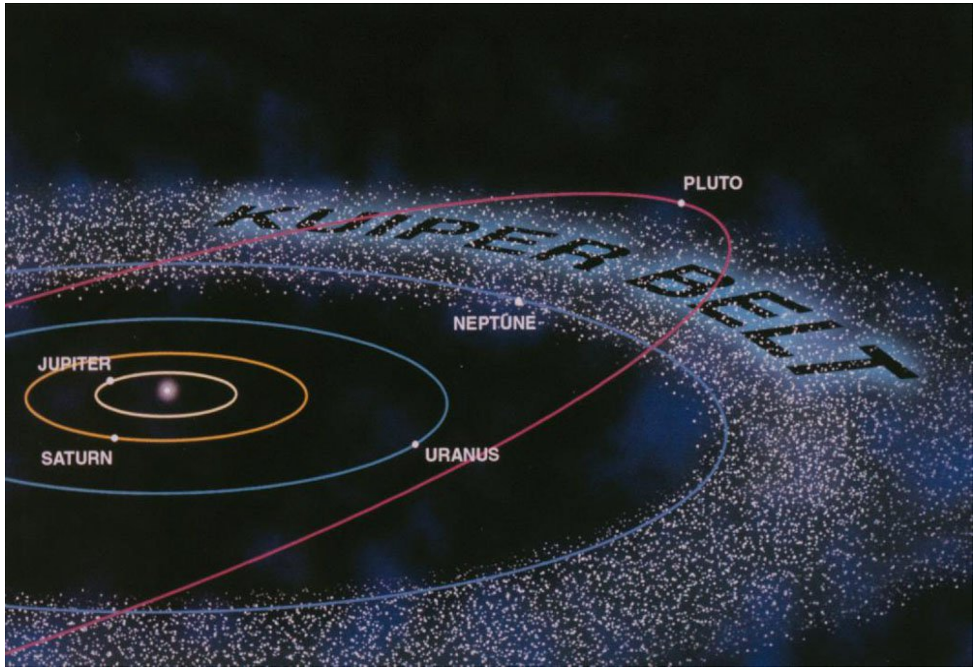
Analysis by La Sapienza/INPOP group (Durante et al. 2020, Iess et al. 2019, Di Ruscio et al 2020)

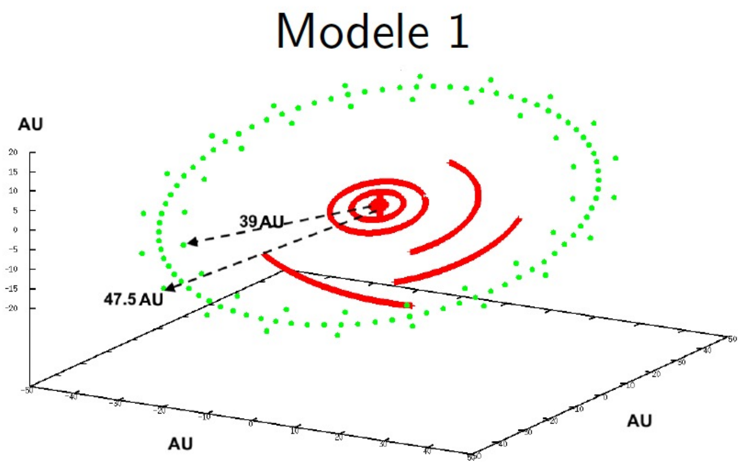


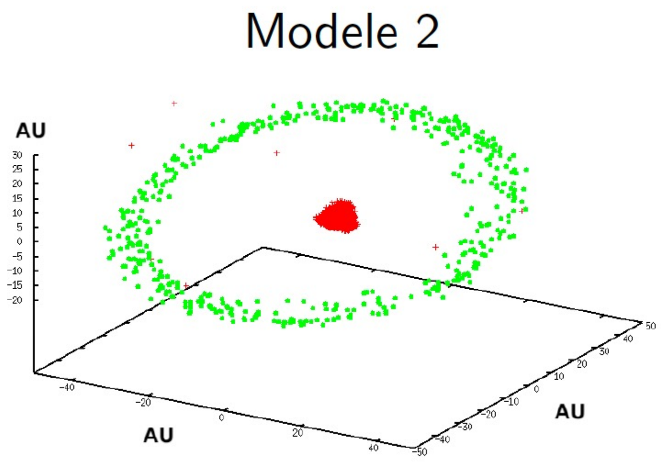


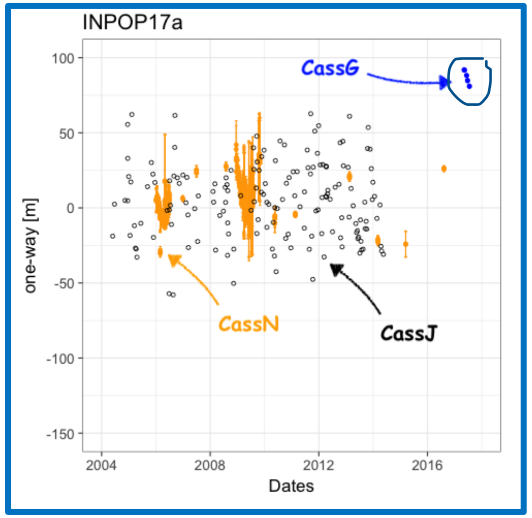
Introduction of new perturbers = Kuiper Belt objects

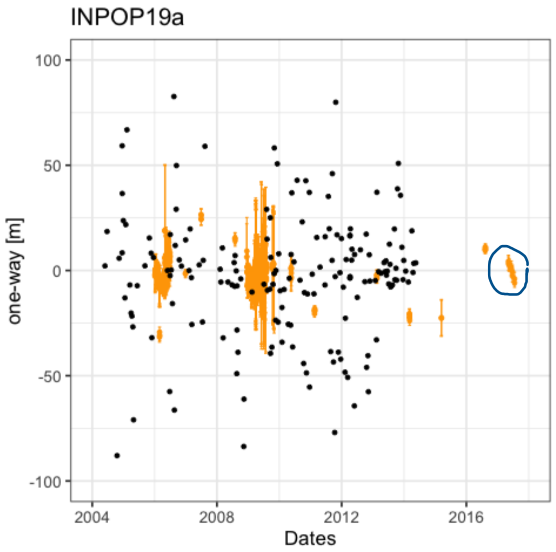
Kuiper Belt objects

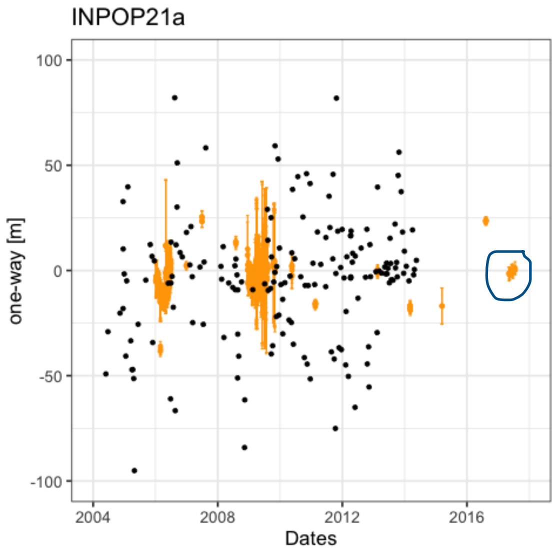






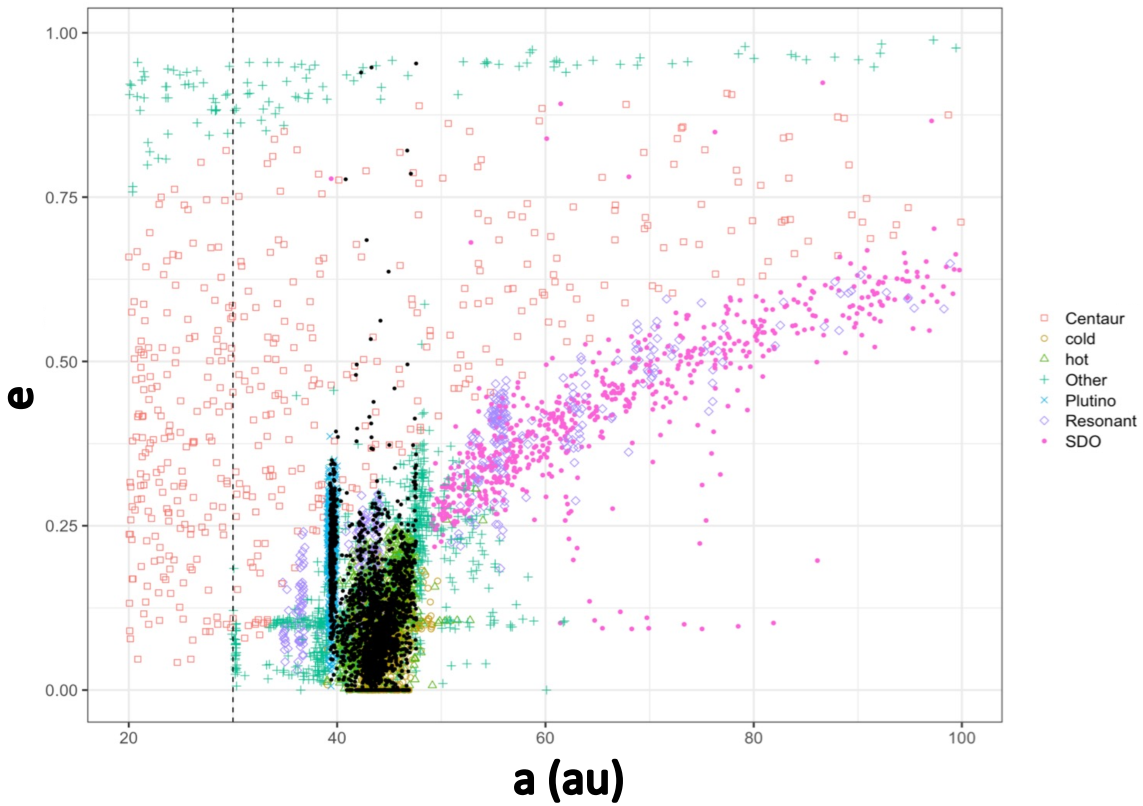






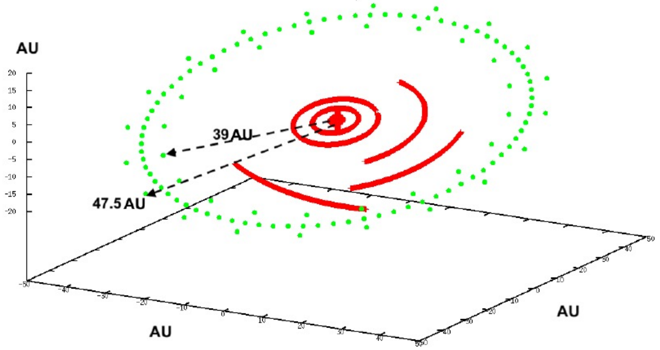
Kuiper Belt objects

- KBO affecting mainly Cassini (Saturn) residuals
- Since INPOP21a : 500 objects randomly selected
- Since INPOP22a : orbit angles are also considered
- With INPOP25c : Plutino (14%) + others (86%)

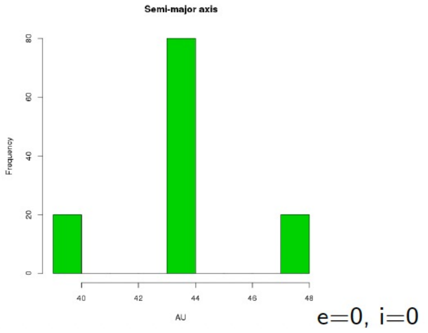


INPOP19a

Modele 1

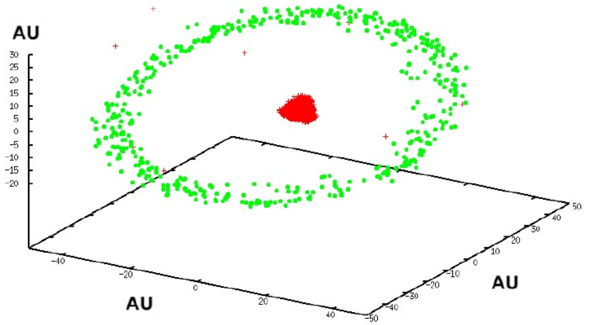


$M_{\text{ring}} = (0.061 \pm 0.001) M_{♁}$

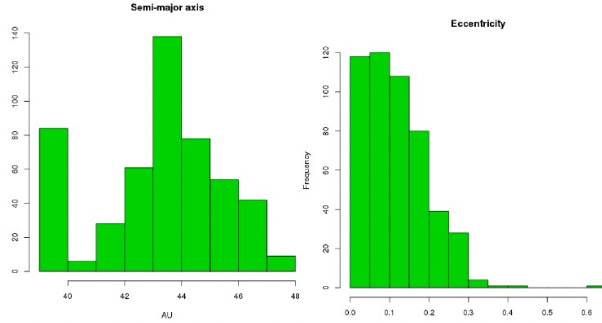


INPOP21a

Modele 2



$M_{\text{ring}} = (0.041 \pm 0.001) M_{♁}$



	GM_KBO [$ua^3 d^{-2}$]	M_KBO
INPOP22a	3.7×10^{12}	$0.004\ M_{\oplus}$
INPOP25c	2.7×10^{12}	$0.003\ M_{\oplus}$

Still KBO mass is smaller than expected !

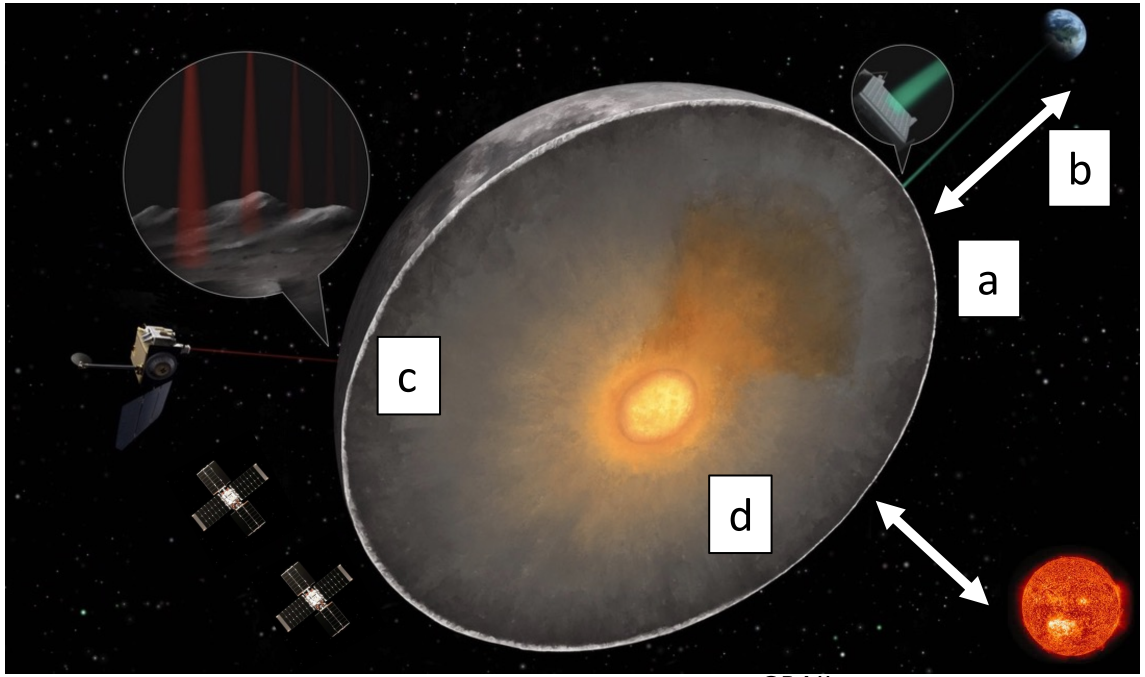
- moon

Dynamical modeling beside GRT : Moon internal structure

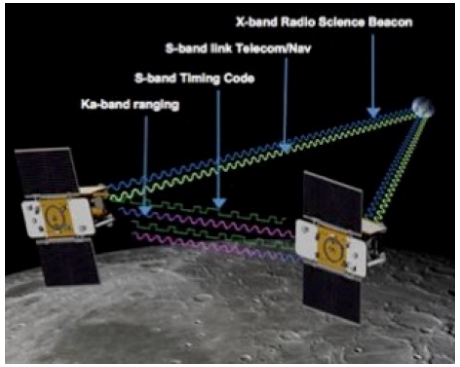
Earth-Moon torques with:

- a — Orbital and rotational coupling (time-varying Euler angle)
- c — Moon surface and mantle deformations
- d — Fluid core (axisymmetric) interaction with the mantle
- Dissipation of energy at the CMB with viscous friction
- b — Impact of Moon tides on Earth orbit and rotation (orbital and rotational time delays)

→ 1 cm accuracy on the earth-moon distances



GRAIL



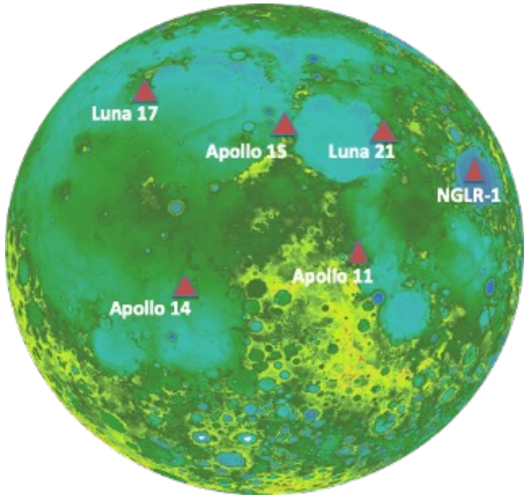
Observational and external constraints:

- 40 years of LLR
- Precise station localization (solid tides, atmospheric and ocean loadings, plate tectonics)
- Gravity fields of Moon and Earth provided by GRAIL and GRACE
- GRAIL = 30 km resolution for a 1200 degree GF*

(Viswanathan et al. 2018, Viswanathan et al 2019)

Challenges for Moon

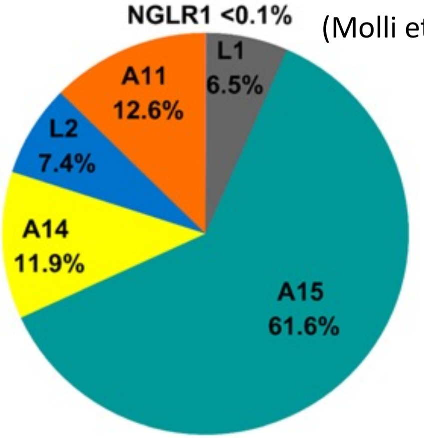
Only 6 control points



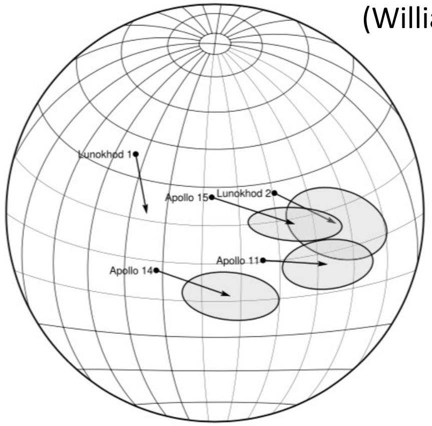
Non-homogeneous distribution of data in space and time

from 04.1970 to 05.2025

(Molli et al. 2026)



(Williams and Boggs 2021)

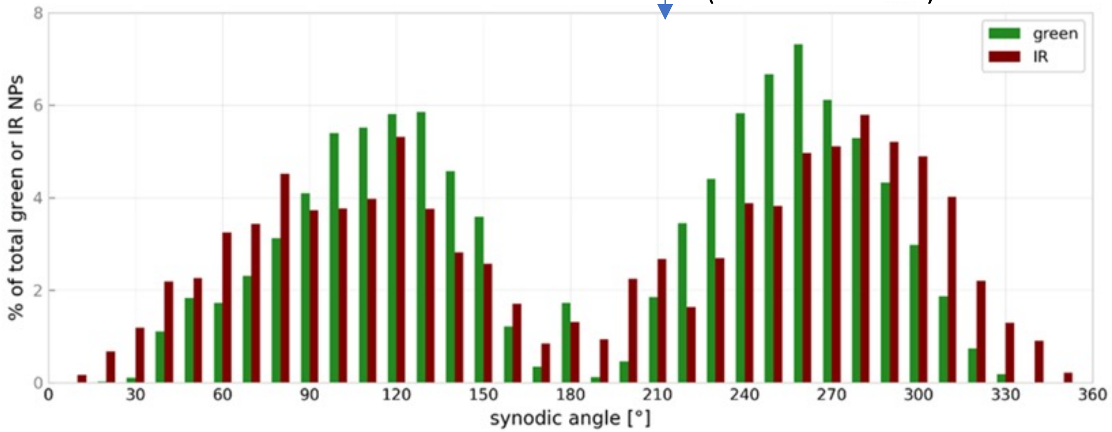


Displacements of LLRR (20 cm in 25 years)

Lacks in the rotational modeling

Inconsistent surface deformation / LRO and GF / GRAIL

(Molli et al. 2026)

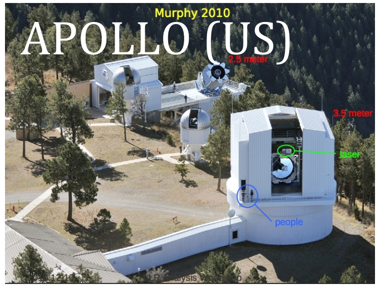


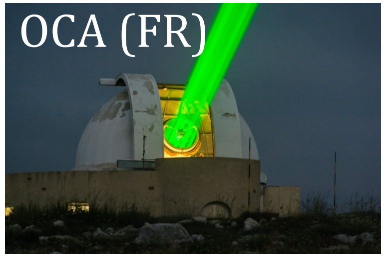
Empirical terms added in orientation angles

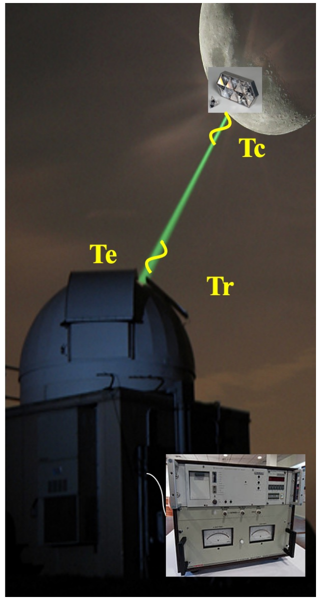
Periods in days	31	206	365	1095	1306	1643	2190
DE440	x	x	x	x	x	x	x
DE430		x	x	x			
INPOP19a		x	x	x			
EMP2021		x	x	x			

Challenges for Moon

Only distances are observed

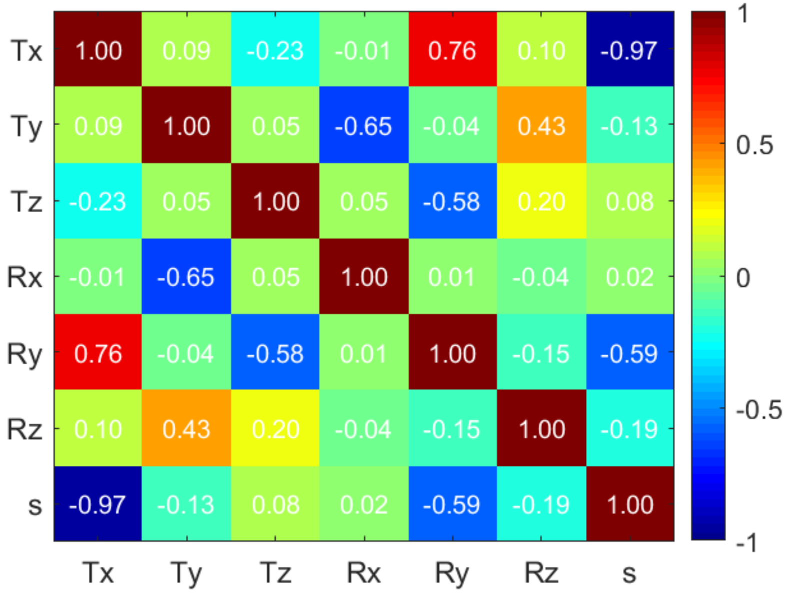






Lacks in the reference frame definition

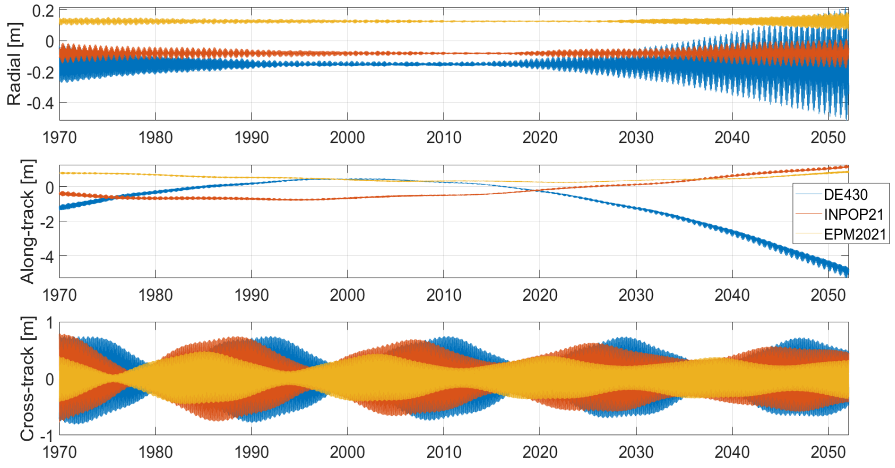
Correlation in S,Tx for Helmert RF definition



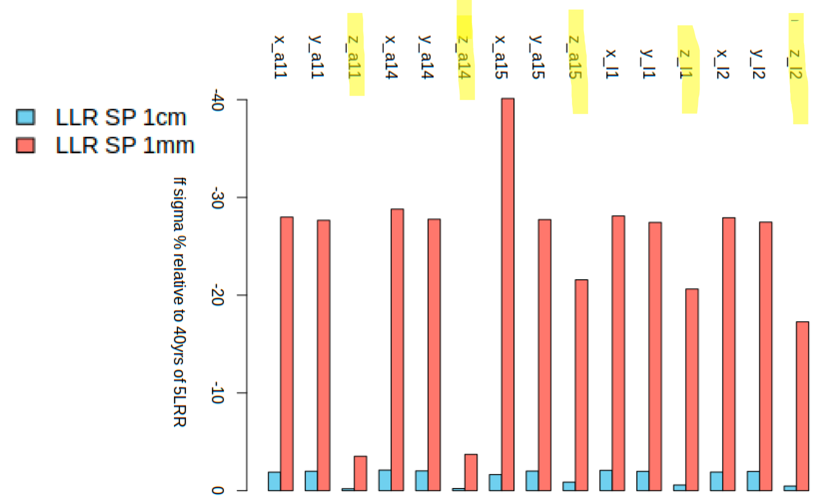
(Sosnika et al. 2025)

Origin is only well constrained along the radial direction

(Sosnika et al. 2025)

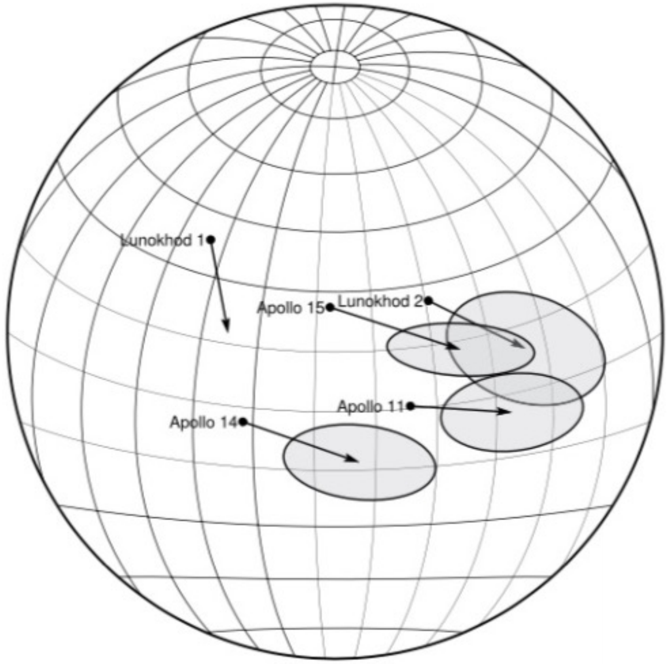


Z-coordinates of LLRR badly estimated



1 cm but still some open questions …

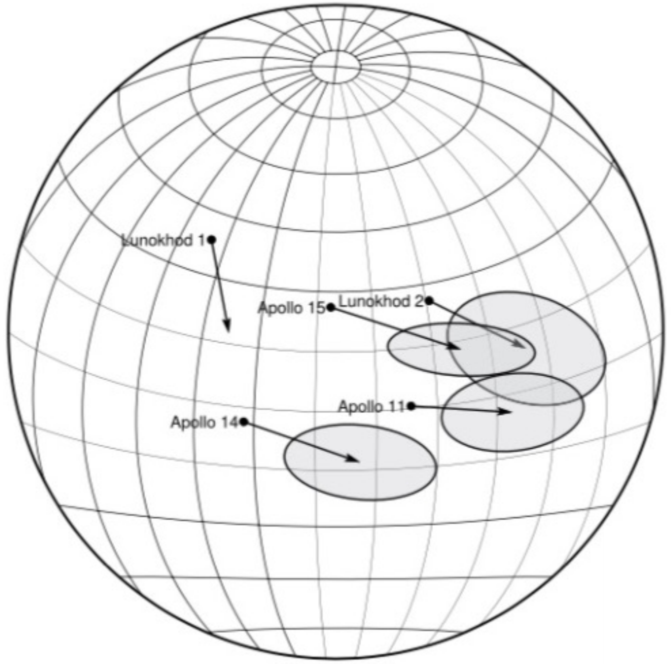
(adapted from Shapiro et al. 2023)



A. LLR RR displacement : 20 cm in 25 years

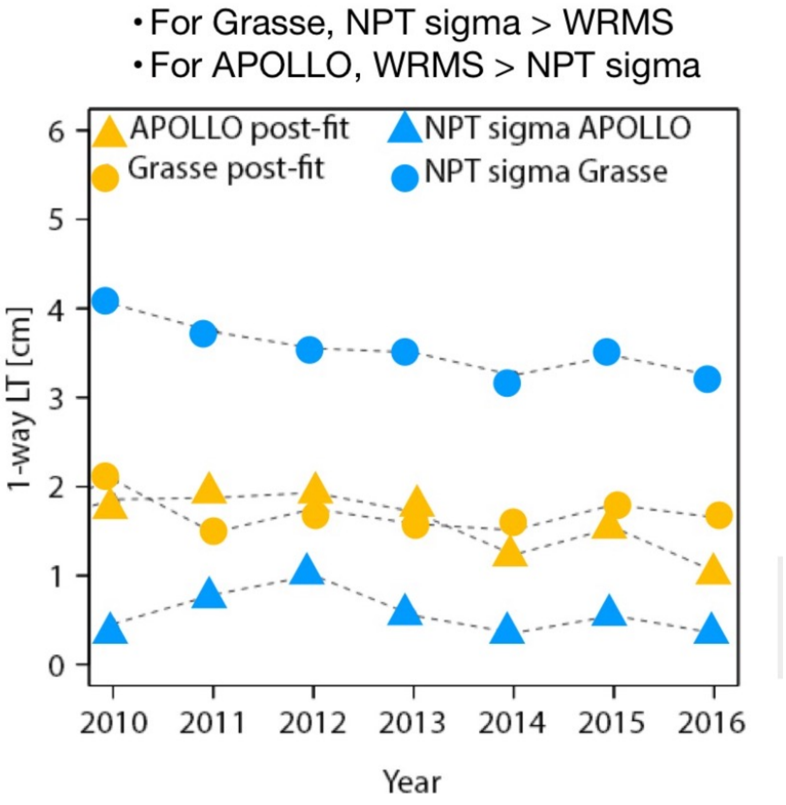
1 cm but still some open questions …

(adapted from Shapiro et al. 2023)



A. LLR RR displacement : 20 cm in 25 years

B. Still models do not represent APOLLO station accuracy



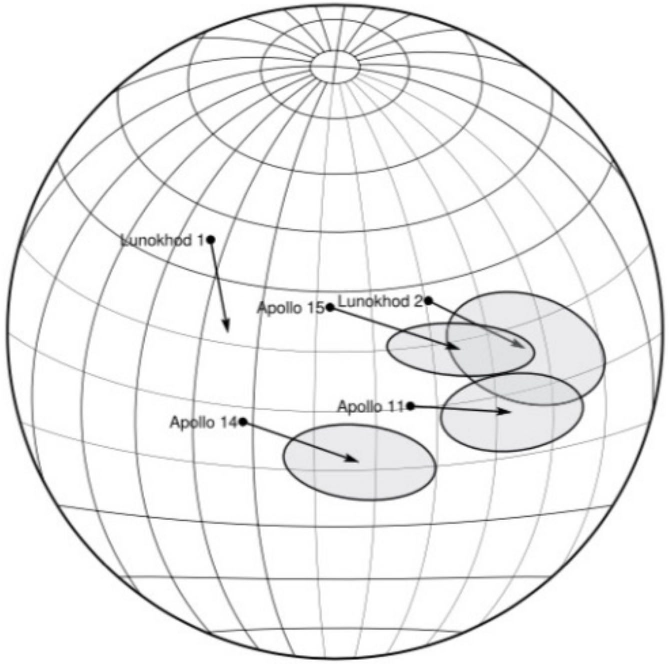
1 cm but still some open questions …

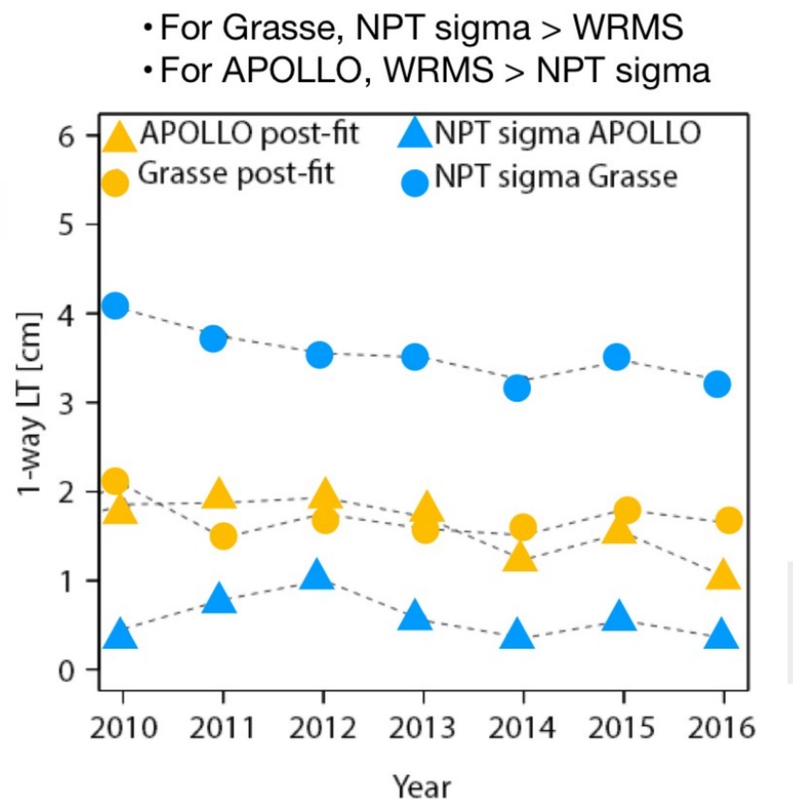
(adapted from Shapiro et al. 2023)

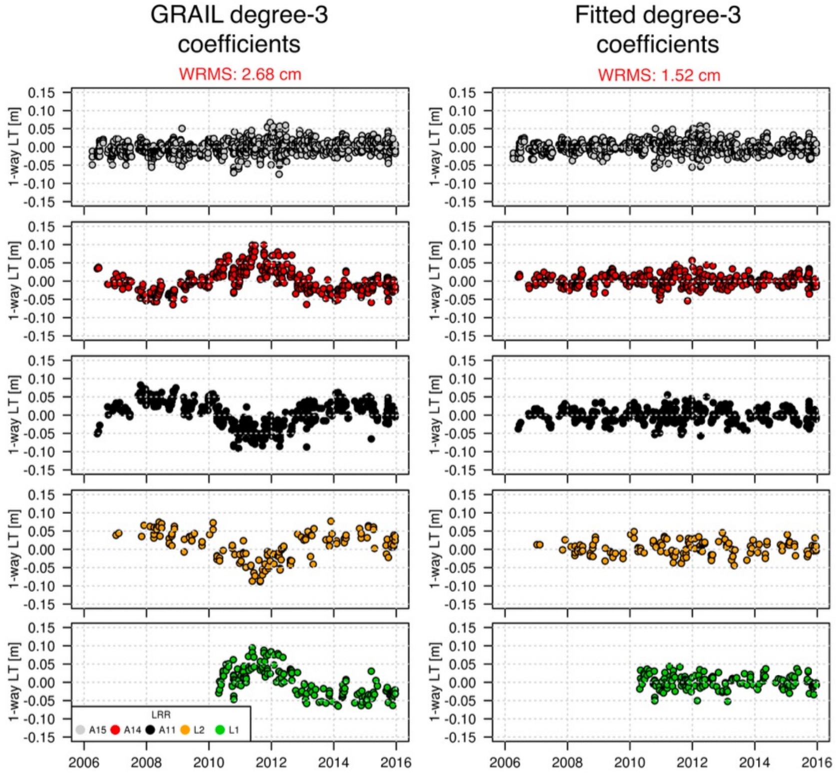
A. LLR RR displacement : 20 cm in 25 years

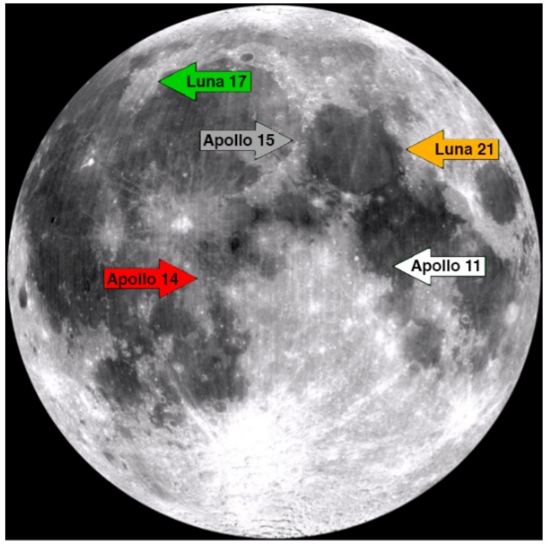
B. Still models do not represent APOLLO station accuracy

C. Still some mismodeling in libration (GF terms, empirical terms)



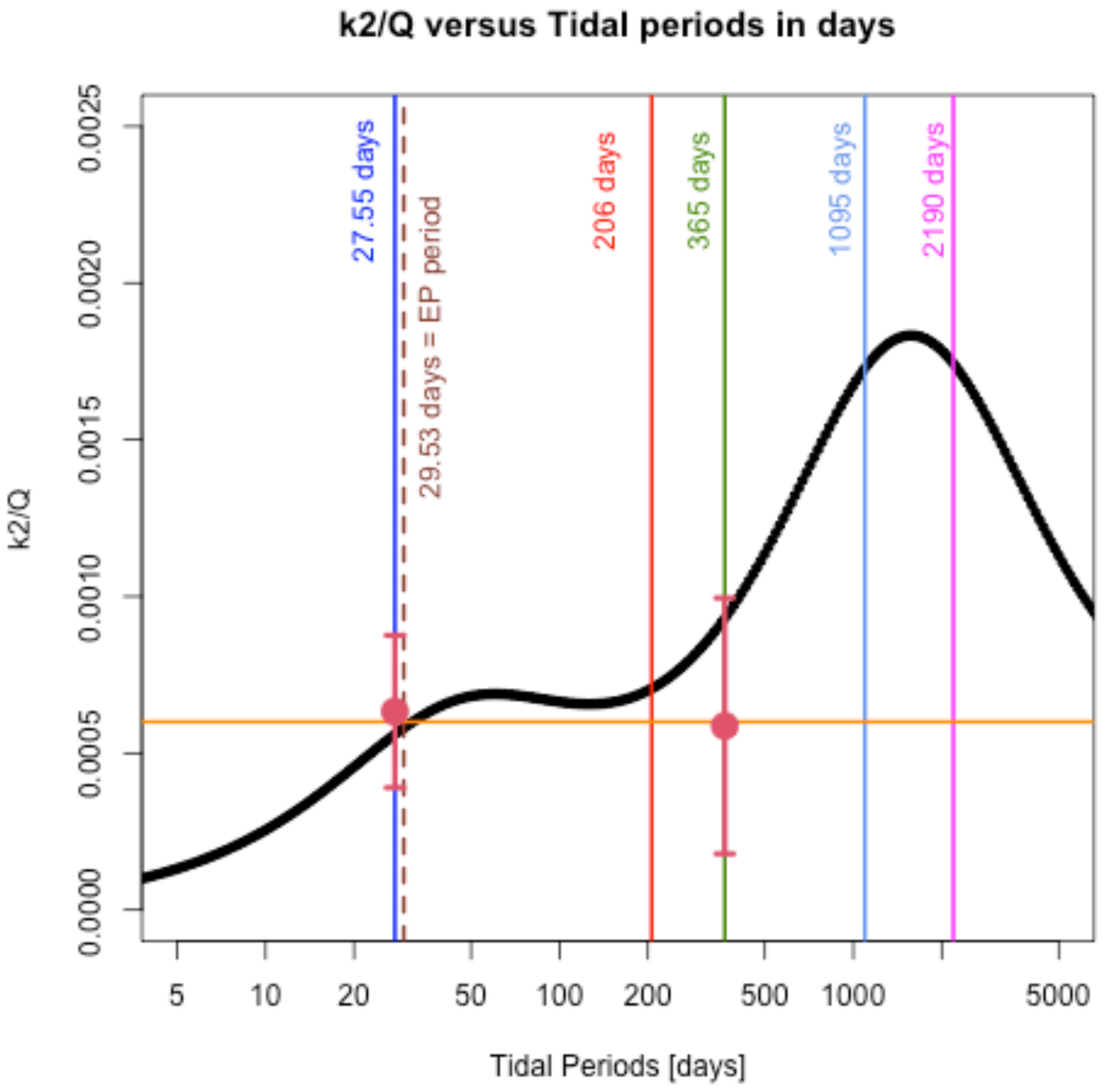






1 cm but still some open questions …

A. LLR RR displacement : 20 cm in 25 years
B. Still models do not represent APOLLO station accuracy
C. Still some mismodeling in libration (GF terms, empirical terms)
D. Degeneracy with Moon internal structure modeling



1 cm but still some open questions …

A. LLR RR displacement : 20 cm in 25 years
B. Still models do not represent APOLLO station accuracy
C. Still some mismodeling in libration (GF terms, empirical terms)
D. Degeneracy with Moon internal structure modeling
E. Empirical terms ! necessary to fit LLR data

Empirical terms added in orientation angles

Periods in days	***31***	***206***	***365***	***1095***	***1306***	***1643***	***2190***
DE440	*x*	*x*	*x*	*x*	*x*	*x*	*x*
DE430		*x*	*x*	*x*			
INPOP19a		*x*	*x*	*x*			
EMP2021		*x*	*x*	*x*