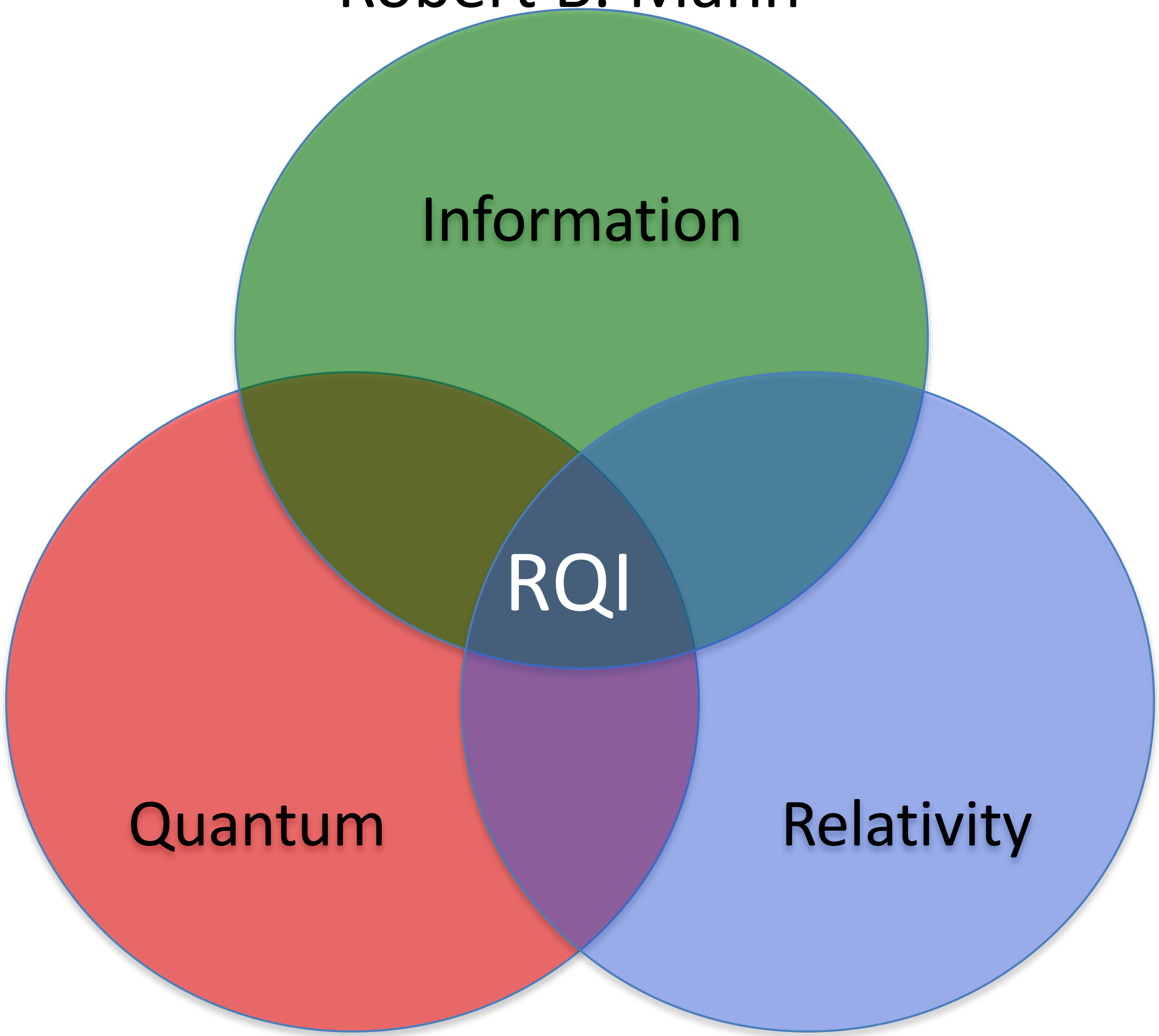
University of Waterloo

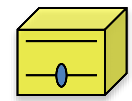
Relativistic Quantum Information

Robert B. Mann



Recall: Quantum Detector Formalism

$$S = -\int d^4x \sqrt{-g}\left[R + \frac{1}{2}\partial_\mu \Phi(x)\partial^\mu \Phi(x) - \xi R \Phi^2(x)\right.$$

$\Omega \updownarrow$  UdW detector

$\rightarrow$ quantum dot

$$\left. + \sum_D \int d\tau_D \frac{m_0}{2}\left[\dot{Q}_D^2(\tau_D) - \Omega_D^2 Q^2\right] + \sum_D \lambda_D \int d^4x Q_D(\tau)\Phi(x)\delta^4\left(x^\mu - z_D^\mu(\tau)\right)\right\}$$

$$H_{ID}(\tau) = \chi_D(\tau)\left[e^{i\Omega_D \tau}\sigma_D^+ + e^{-i\Omega_D \tau}\sigma_D^-\right]\Phi[z_D(\tau)]$$

switcher

monopole operator

$$\chi_D = \exp\left(-\frac{(\tau - \tau_D)^2}{2\sigma_D^2}\right)$$

$$\sigma_D^+ := |1_D\rangle\langle 0_D| = |e_D\rangle\langle g_D|$$

$$\sigma_D^+ := |0_D\rangle\langle 1_D| = |g_D\rangle\langle e_D|$$

Initial state

$$\rho_I = \rho_{D_I}\rho_{\Phi_I}$$

Unitarily evolve

$$U = \mathcal{T} e^{-i\int dt\left[\sum_D \frac{d\tau_D}{dt} H_{ID}(\tau_D)\right]}$$

Final state

$$\rho_F = U\rho_I U^\dagger$$

Trace over field

$$\rho_{D_F} = \mathrm{Tr}_\Phi\left[\rho_F\right]$$

single detector

$$\rho_D = \begin{pmatrix} 1 - P_D & 0 \\ 0 & P_D \end{pmatrix}$$

response

two detectors

$$\rho = \begin{pmatrix} 1-P_A-P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

entanglement, mutual information

$$P_D = \lambda^2 \int_{-\infty}^{\infty} d\tau_D \int_{-\infty}^{\infty} d\tau_{D'}\, \chi_D(\tau_D)\chi_{D'}(\tau_{D'}) e^{-i\Omega_D(\tau_D - \tau_{D'})} W\left(x_D(\tau_D), x_D(\tau_{D'})\right) \quad D = A, B$$

response

$$C = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A - \Omega_B\tau_B)} W\left(x_A(t), x_B(t')\right)$$

correlations

$$X = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \Big[\frac{d\tau_A}{dt'}\frac{d\tau_B}{dt} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_B\tau_B + \Omega_A\tau_A)} W\left(x_A(t'), x_B(t)\right)$$
$$+ \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A + \Omega_B\tau_B)} W\left(x_B(t'), x_A(t)\right) \Big]$$

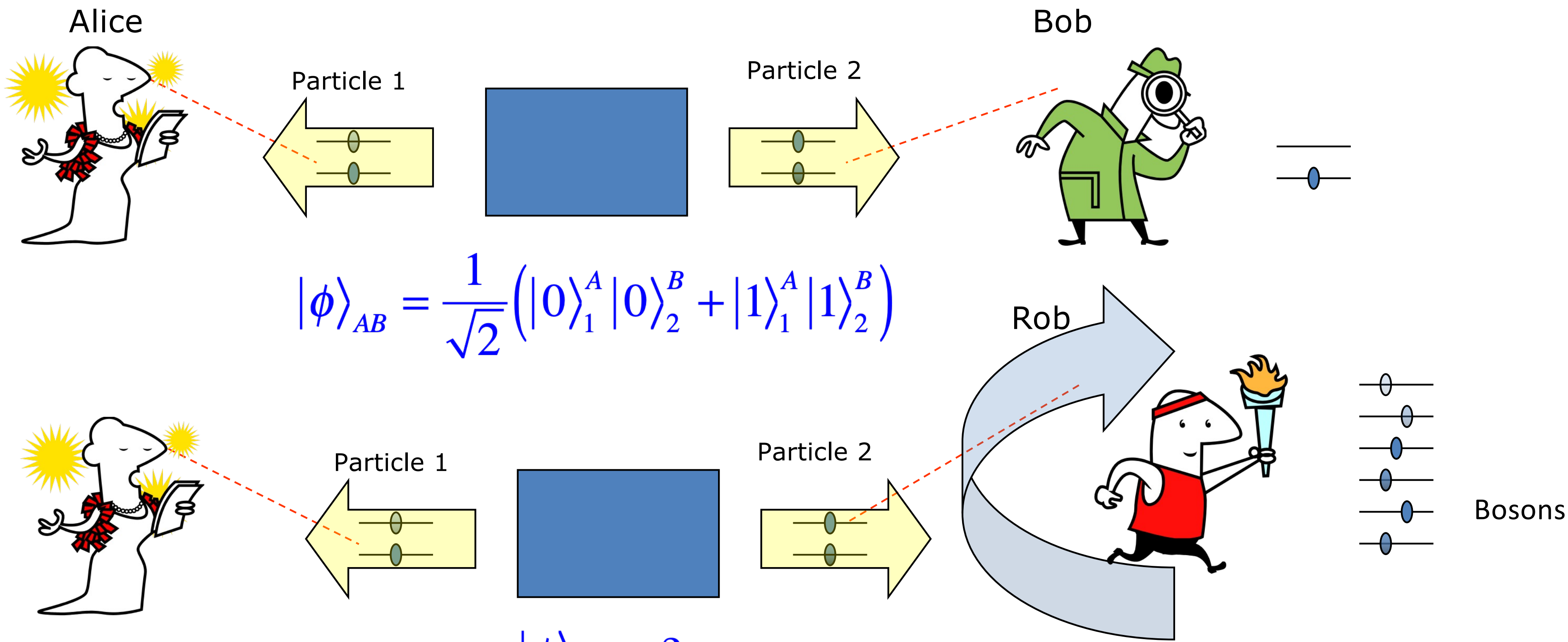
non-local correlations

Wightman Function

$$W(x, x') := \langle 0 | \phi(x)\phi(x') | 0 \rangle \qquad \tau_D = \gamma_D t$$

- detector motion
- spacetime curvature
- initial field state (vacuum, thermal, coherent, etc)
- choice of quantum field (scalar, vector, ...)

RQI Phenomena



$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A |0\rangle_2^B + |1\rangle_1^A |1\rangle_2^B\right)$$

$$|\phi\rangle_{AB} = ?$$

- Acceleration-induced Temperature
- Entanglement Degradation
- Entanglement Harvesting
- Spacetime Superposition

Single Detector Response

$$\rho_A = \mathrm{Tr}_B \rho = \begin{pmatrix} 1-P_A & 0 \\ 0 & P_A \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$W(x,x') \coloneqq \langle 0|\phi(x)\phi(x')|0\rangle$$

$$H_{\text{int}} = c\chi(\tau)\mu(\tau)\phi(x(\tau))$$

$$P(E) = c^2 \left|\langle 0_d|\mu(0)|E\rangle\right|^2 \mathcal{F}(E)$$

$$\mathcal{F}(E) = \Re\left[\int_{-\infty}^{\infty} du\,\chi(u)\int_0^{\infty} ds\,\chi(u-s)e^{-iEs}W^+(u,u-s)\right]$$

$$\frac{d\mathcal{F}}{d\tau}(E;M,\ell,\ldots) = \frac{1}{4} + 2\Re\left[\int_0^{\Delta\tau} ds\,e^{-iEs}W^+(\tau,\tau-s)\right]$$

Detector Response Rate

Thermalize

EDR Ratio

$$\mathcal{R} = \frac{\dot{\mathcal{F}}(\Omega)}{\dot{\mathcal{F}}(-\Omega)} = e^{-\Omega/T_{EDR}}$$

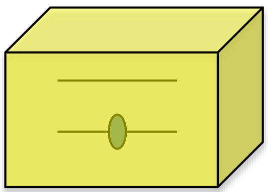
$$T_{EDR} = -\frac{\Omega}{\log \Re}$$

Detector temperature

Oscillating Vacuum Detectors

Non-uniform acc'n:
Ostapchuk/Lin/ RBM/Hu
JHEP 1207 (2012) 072

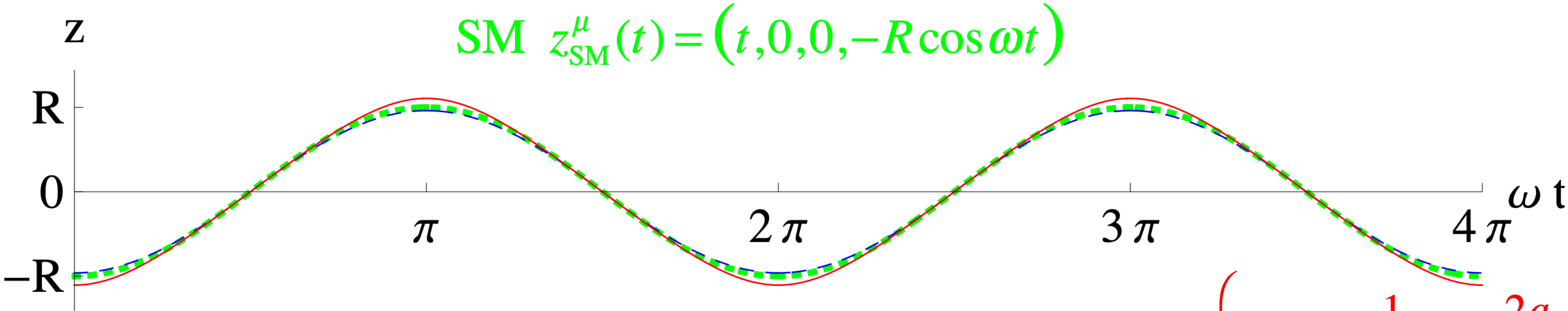
$$S = -\int d^4x \sqrt{-g}\, \frac{1}{2} \partial_\mu \Phi(x) \partial^\mu \Phi(x)$$



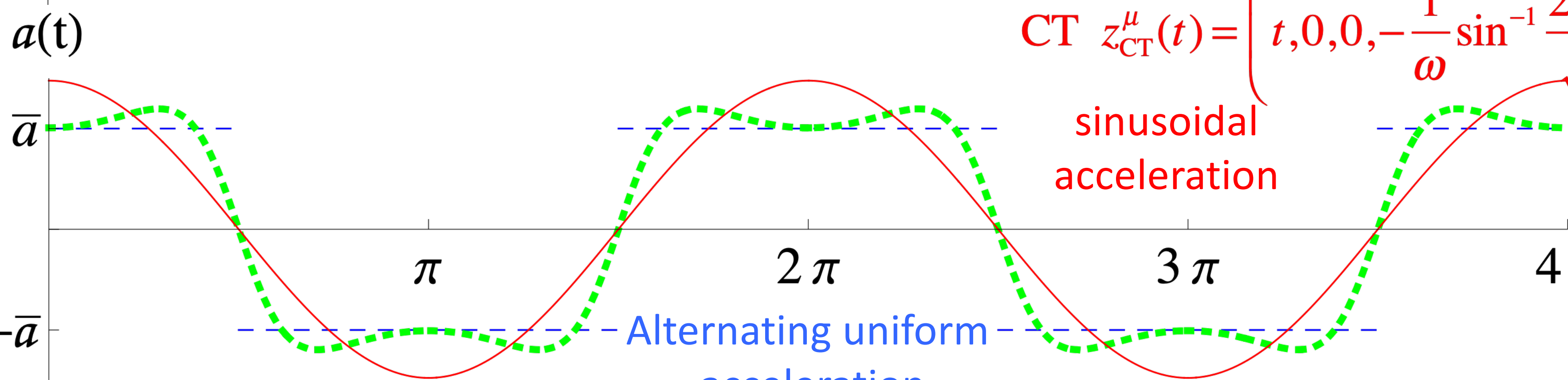
$$+ \int d\tau \left\{ \frac{m_0}{2} \left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2 \right] + \lambda_0 \int d^4x\, Q(\tau) \Phi(x) \delta^4 \left(x^\mu - z^\mu(\tau) \right) \right\}$$

sinusoidal motion

$$\text{SM} \quad z^\mu_{\text{SM}}(t) = \left(t, 0, 0, -R\cos\omega t \right)$$



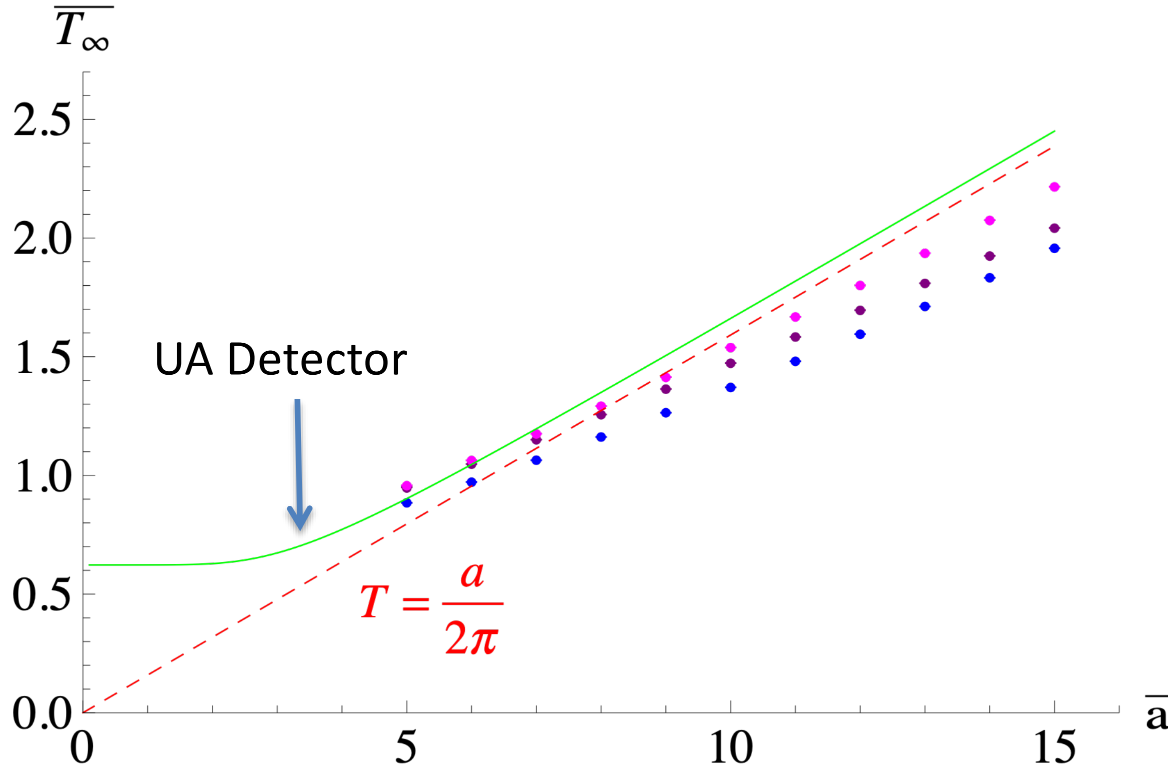
$$\text{CT} \quad z^\mu_{\text{CT}}(t) = \left(t, 0, 0, -\frac{1}{\omega} \sin^{-1} \frac{2a_0 \cos\omega t}{\sqrt{1+4a_0^2}} \right)$$

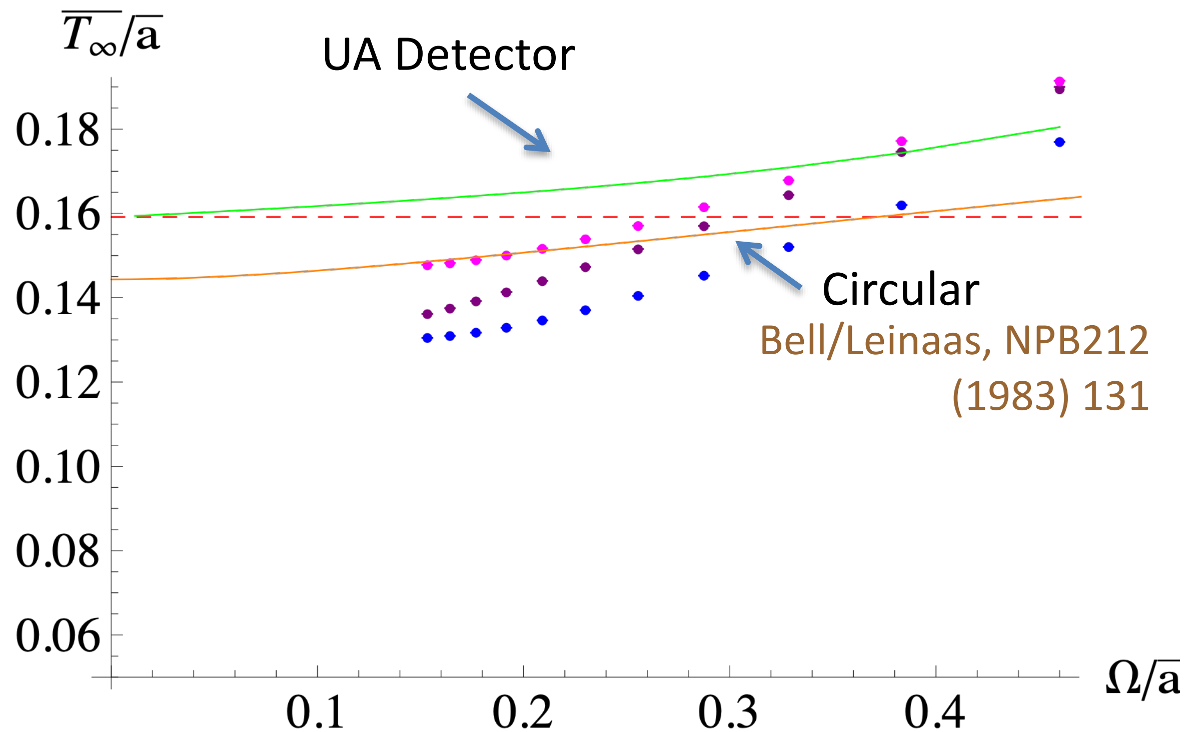


$$\text{AUA} \quad z^\mu_{\text{AUA}}(\tau) = \left(\frac{1}{a} \left[\sinh a \left(\tau - \frac{n\tau_p}{2} \right) + 2n \sinh \frac{a\tau_p}{4} \right], 0, 0, \frac{(-1)^n}{a} \left[\cosh a \left(\tau - \frac{n\tau_p}{2} \right) + \left\{ (-1)^n - 1 \right\} \cosh \frac{a\tau_p}{4} \right] \right)$$

Results

Doukas/Lin/Hu/RBM JHEP 1311 (2013) 119





Effective Temperature

$$T_{\rm eff}(\tau) = \left[\frac{k_B}{\hbar\Omega_r}\ln\left(\frac{U(\tau)+\hbar/2}{U(\tau)-\hbar/2}\right)\right]^{-1} \to T_{\rm eff}(\infty) = \bar{T}_\infty$$

$$U(\tau) \equiv \sqrt{\langle\hat{P}^2(\tau)\rangle\langle\hat{Q}^2(\tau)\rangle - \langle\hat{Q}(\tau),\hat{P}(\tau)\rangle^2}$$

Average Acceleration $\quad \bar{a} \equiv \frac{\int_P a(\tau)d\tau}{\int_P d\tau}$

CT worldline

SM worldline

AUA worldline

$$\omega = 20$$

$$\gamma \equiv \lambda_0^2 / 8\pi m_0 = 0.01$$

$$\Omega \equiv \sqrt{\Omega_r^2 - \gamma^2} = 2.3$$

$$\Lambda = -\ln\frac{\hbar\Omega_R}{E_{\rm cutoff}} = 20$$

Accelerating Detectors Get Hot!

$$P \rightarrow \text{thermal} \qquad T = \frac{a}{2\pi}\left(\frac{\hbar}{k_B c}\right)$$

S.A. Fulling PRD7 (1973) 2850 P.C.W. Davies J Phys A8 (1975) 609
W. G. Unruh PRD14 (1976) 3251

- A single detector, accelerating uniformly forever, will respond as though it is in a heat bath
- Idealized conditions can be relaxed
  - Finite time acceleration
  - Motion within cavities
  - Non-uniform motion
- A cold vacuum to one observer is a hot vacuum to another
- Effect is robust

Can even turn this into a heat engine!

Arias/ Oliveira/ Sarandy JHEP **18** (2018) 168
F. Gray/RBM JHEP **11** (2018) 174

Detector Response Outside Black Holes

- BTZ Black holes
  - Static and Rotating
- Schwarzschild Black Holes
- Schwarzschild AdS Black Holes
- All for various boundary conditions, detector trajectories

Hodgkinson/Louko PRD86 (2012) 064031

Hodgkinson/Louko/Ottewill PRD89 (2014) 104002

Ng/Hodgkinson/Louko/ RBM/Martin-Martinez PRD90 (2014) 064003

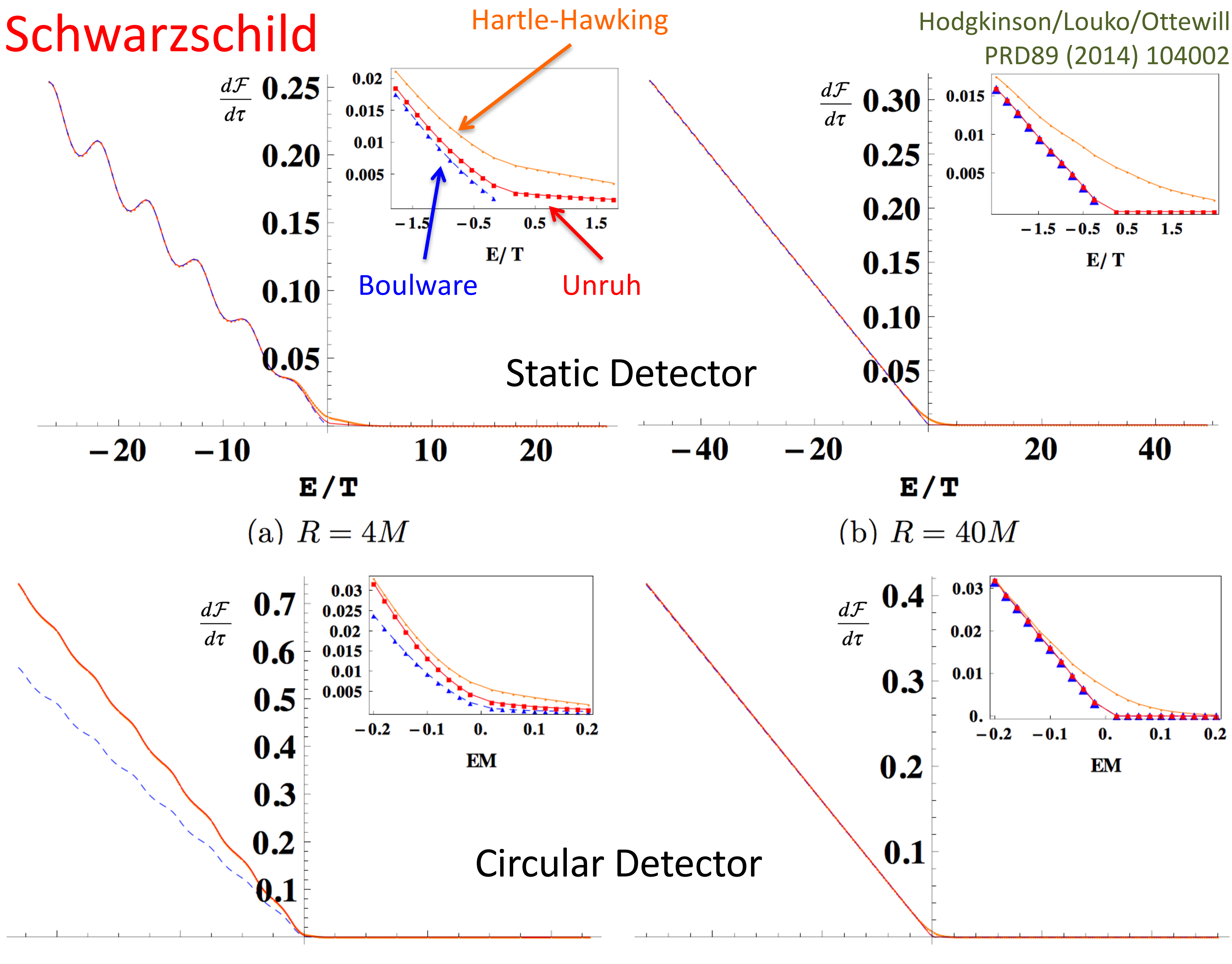
$$H_{\text{int}} = c\underbrace{\chi(\tau)}_{\text{switch}}\underbrace{\mu(\tau)}_{\text{monopole operator}}\overbrace{\phi(x(\tau))}^{\text{field}}$$

$$P(E) = c^2 \left|\langle 0_d | \mu(0) | E\rangle\right|^2 \mathcal{F}(E)$$

$$\mathcal{F}(E) = \Re\left[\int_{-\infty}^{\infty} du\,\chi(u)\int_0^{\infty} ds\,\chi(u-s)e^{-iEs}\overbrace{W(u,u-s)}^{\text{Wightman function}}\right]$$

$$\frac{d\mathcal{F}}{d\tau}(E;M,\ell,\ldots) = \frac{1}{4} + 2\Re\left[\int_0^{\Delta\tau} ds\,e^{-iEs}W(\tau,\tau-s)\right]$$

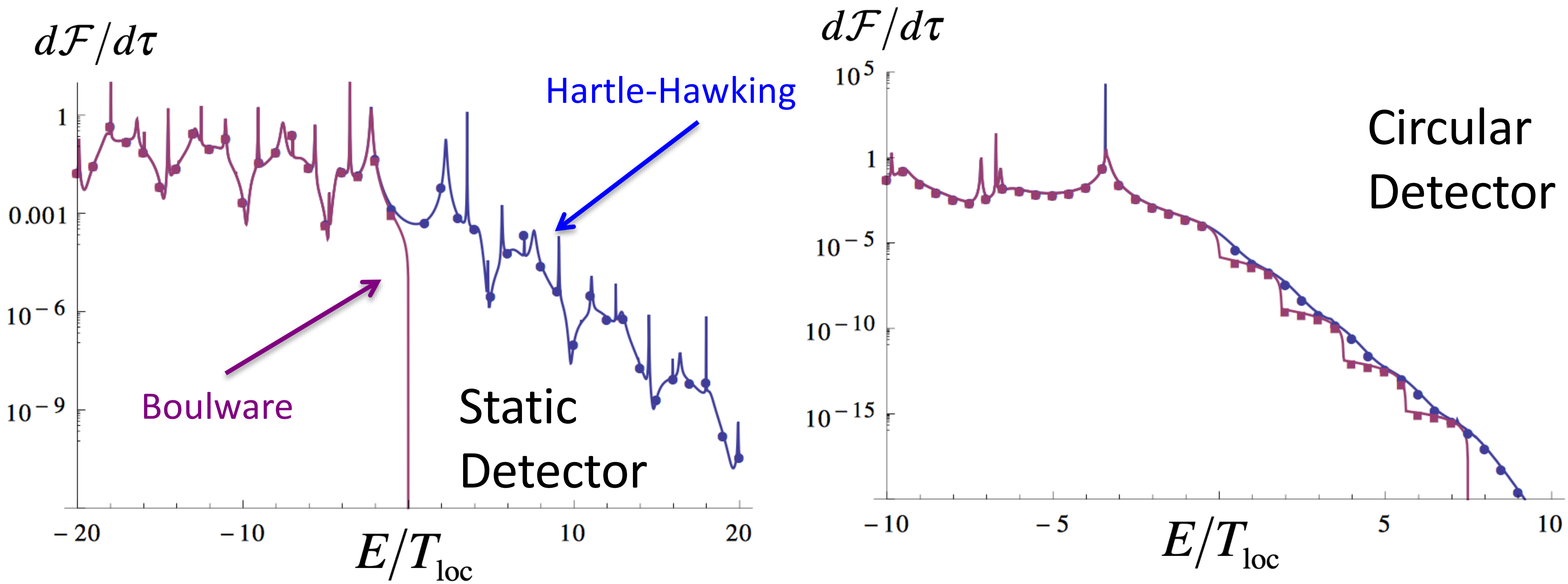
Detector Response Rate



Schwarzschild
Hartle-Hawking
Hodgkinson/Louko/Ottewill
PRD89 (2014) 104002
Boulware
Unruh
Static Detector
Circular Detector
$\frac{d\mathcal{F}}{d\tau}$
E/T
EM
(a) $R = 4M$
(b) $R = 40M$

Schwarzschild-AdS

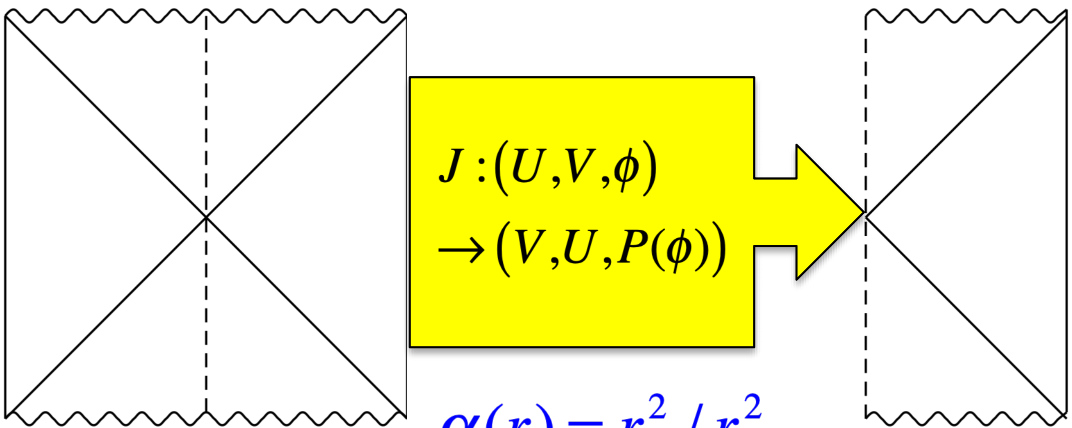
Ng/ Hodgkinson/Louko/RBM/Martin-Martinez
PRD90 (2014) 064003



- Spikes due to Quasinormal mode resonances
- Visible only when black hole is much smaller than AdS length
- Peaks become higher and sharper as black hole size decreases

Looking Inside Black Holes

A. Smith & R.B. Mann
CQG 31 (2014) 082001



$\alpha(r) = r^2 / r_h^2$

BTZ Black Hole

BTZ Geon

$$f(r) = \left(\frac{r^2}{\ell^2} - M\right)$$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2$$

$$= \frac{\ell^2\left[4\mathrm{d}U dV - M(1-UV)^2 d\phi^2\right]}{(1+UV)^2}$$

- No classical way of distinguishing these spacetimes
- Topological features hidden behind horizon
- But quantum fields probe all of spacetime
- Can a UdW detector `look' inside a black hole?

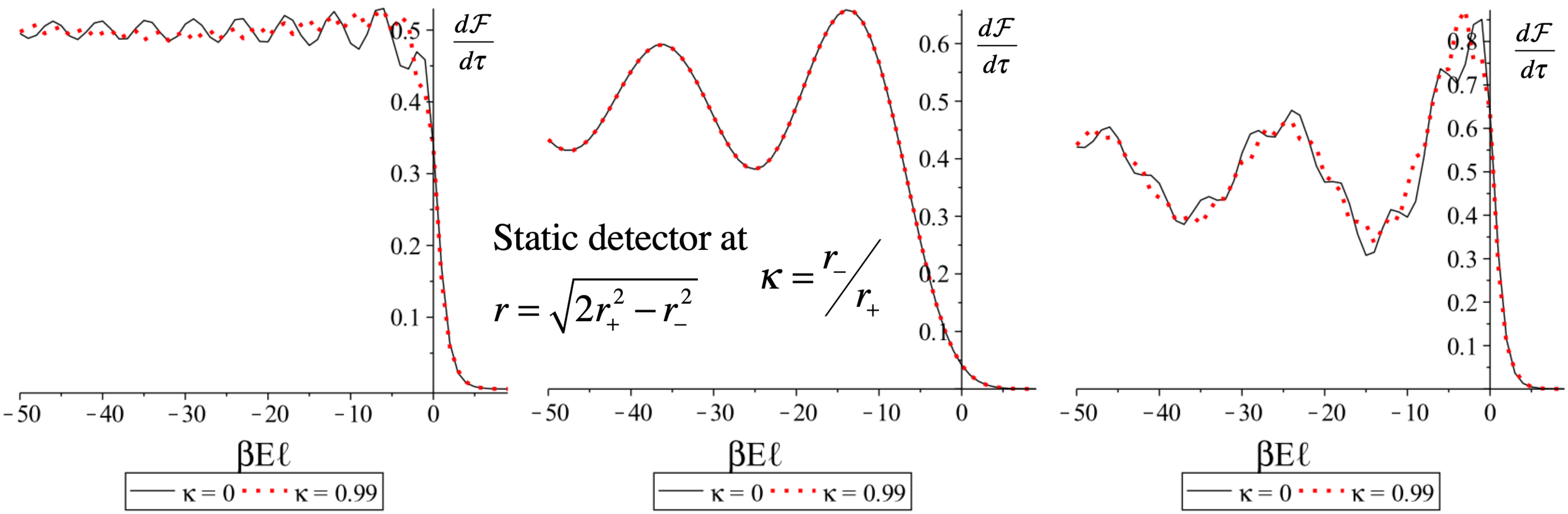
$$\frac{d\mathcal{F}}{d\tau}(E) = \frac{d\mathcal{F}}{d\tau}_{\mathrm{BTZ}}(E) + \Delta\frac{d\mathcal{F}}{d\tau}(E,\tau)_{\mathrm{geon}}$$

BTZ

Hodgkinson/Louko PRD86 (2012) 064031

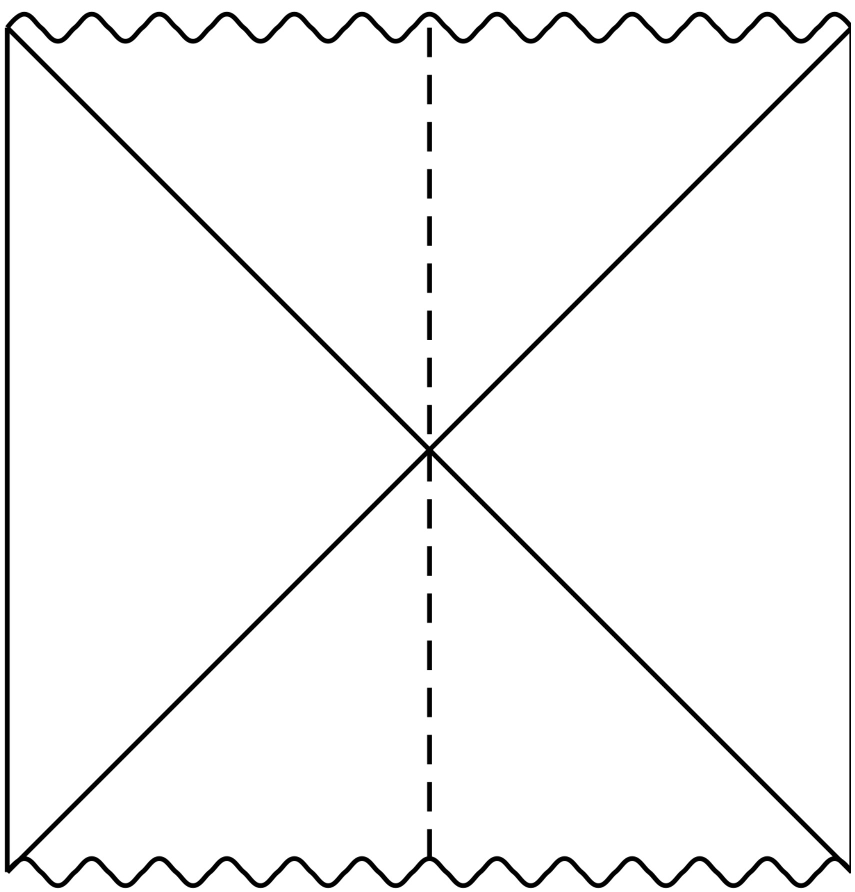
- Wightman function given by image sum not mode sum
- Can consider both rotating black holes and radially infalling detectors

Transparent Dirichlet Neumann



BTZ Black Hole and Geon

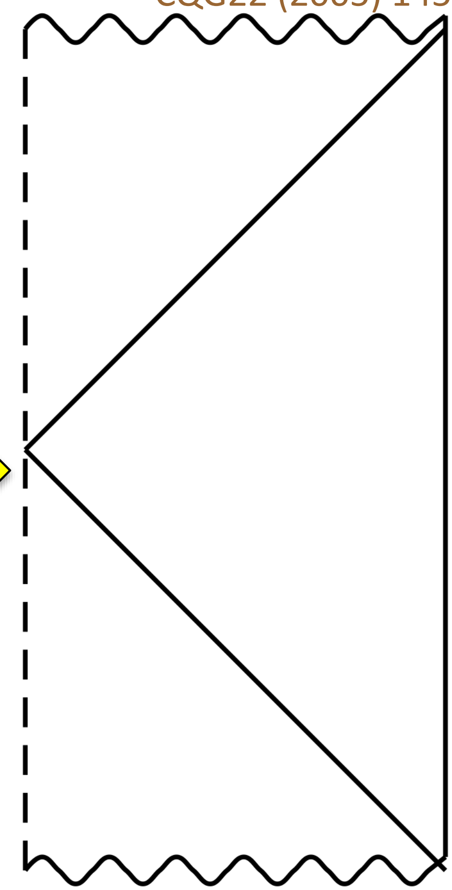
Louko Lect.Notes Phys. 541:188 (2000)
Louko/RBM/Marolf CQG22 (2005) 1451



BTZ Black Hole

$$J:(U,V,\phi) \to (V,U,P(\phi))$$

$$\mathcal{M}_{\text{geon}} = \mathcal{M}_{\text{BTZ}} / \mathbb{Z}_2$$



BTZ Geon

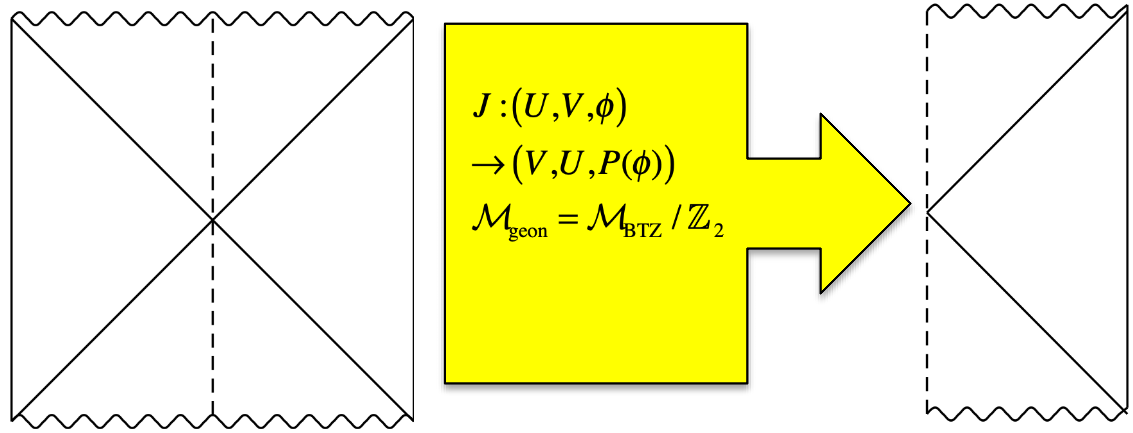
$$ds^2 = -\frac{\ell^2}{(1+UV)^2}\left[-4\,dU\,dV + M(1-UV)^2\,d\phi^2\right]$$

Q: Can we "peek" inside to see what the topological structure is?

A: No, classically

Friedman/Schleich/Witt PRL **71** (1993) 1486
J. Louko and K. Schleich. . Class.Quant.Grav., **16** 2005 (1999)

BTZ BH/Geon Transition Rates



BTZ Black Hole

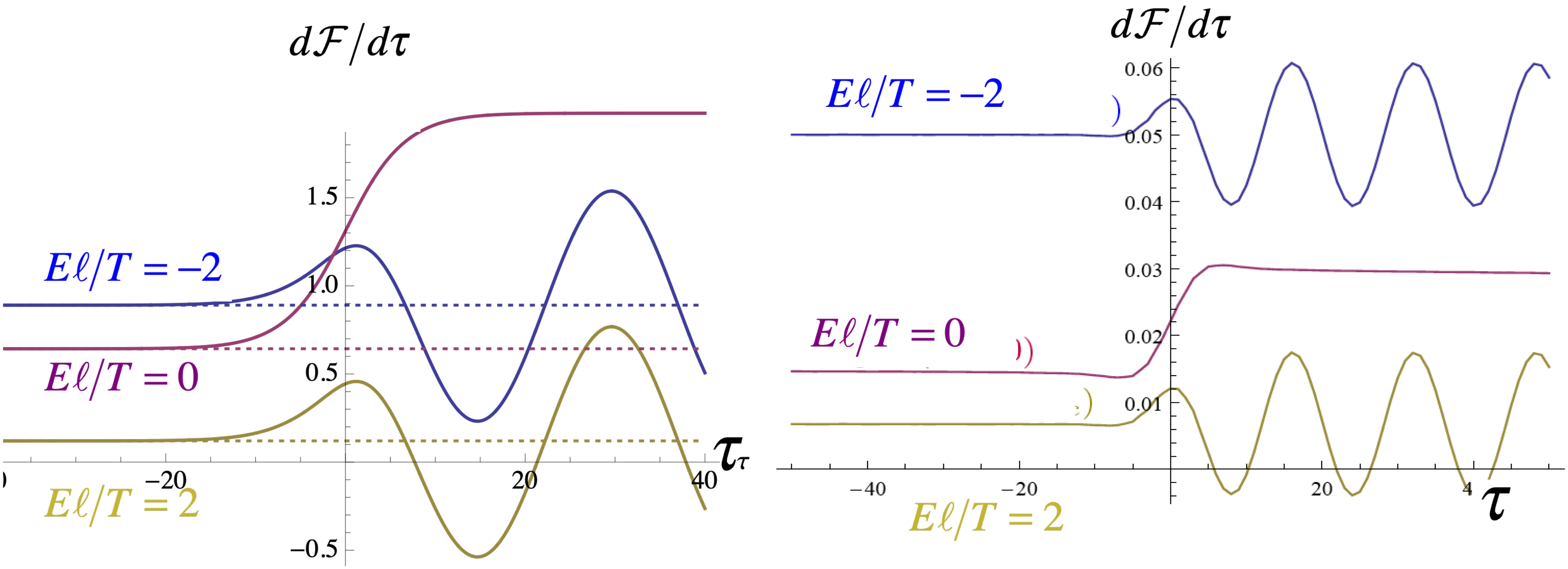
BTZ Geon

$$W_A^{(\zeta)}(x,x')=\frac{1}{4\pi}\left(\frac{1}{\sqrt{\Delta X^2(x,x')}}-\frac{\zeta}{\sqrt{\Delta X^2(x,x')+4\ell^2}}\right)$$

$$W_{\text{BTZ}}(x,x')=\sum_n W_A^{(\zeta)}\left(x,\Lambda^n x'\right)$$

$$W_{\text{geon}}(x,x')=\sum_{m\in\{0,1\}} W_{\text{BTZ}}^{(\zeta)}\left(x,J^m x'\right)=\sum_{m\in\{0,1\}}\sum_n W_A^{(\zeta)}\left(x,J^m\Lambda^n x'\right)$$

$$\dot{\mathcal{F}}_{\tau\text{geon}}(E)=\dot{\mathcal{F}}_{\text{BTZ}}(E)+\Delta\dot{\mathcal{F}}_{\text{geon}}(E,\tau)$$



BTZ Geon

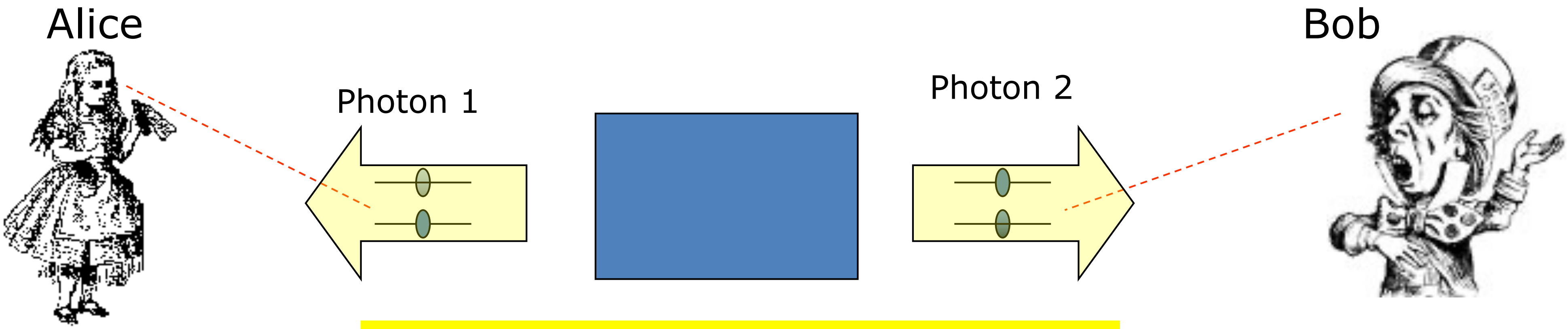
Smith/RBM CQG31 (2014) 082001

Schwarzschild Geon

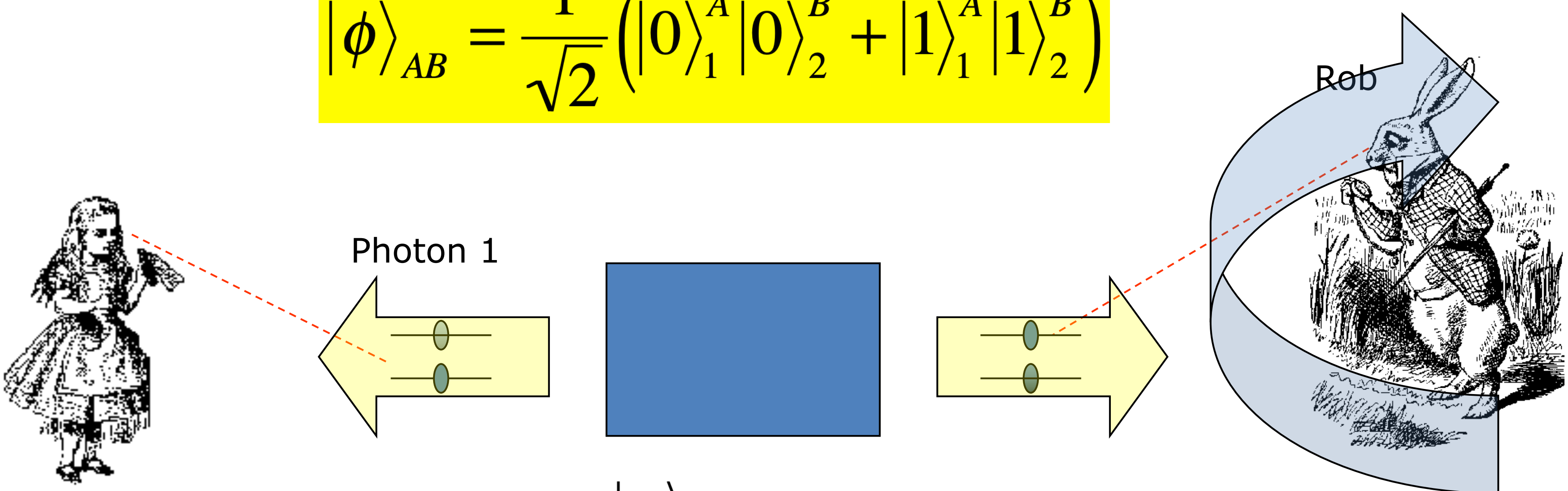
Ng/RBM/Martin-Martinez
PRD**96** (2017) 085004

- Time dependence due to indefinite sign of Killing vector at origin
- Distinction vanishes at large distances and at past/future infinity

Entanglement Degradation in Non-Inertial Frames



$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A |0\rangle_2^B + |1\rangle_1^A |1\rangle_2^B\right)$$



$$|\phi\rangle_{AB} = ?$$

Rindler Vacuum

KG eqn $\nabla^\lambda \nabla_\lambda \varphi - m^2 \varphi = 0$ $\xrightarrow{\text{Minkowski}}$ $\partial_t^2 \varphi - \partial_x^2 \varphi - \partial_{\vec{w}}^2 \varphi - \cancel{m^2}^{\,0} \varphi = 0$

$\Rightarrow$ $\varphi(x) = \int d^{D-1}k \left[\phi_k(x)\, \mathbf{a}_k + \phi_k^*(x)\, \mathbf{a}_k^\dagger \right]$ $\qquad \phi_k(x) = \phi_k(\vec{x}) e^{-i\omega t}$ $\qquad \omega = |\vec{k}|$

$\xrightarrow{\text{Rindler}}$ $\partial_{\mathrm{T}}^2 \phi - \kappa^2 \left(\mathrm{X} \partial_{\mathrm{X}} \right)^2 \phi - \partial_{\vec{W}}^2 \phi = 0$

$\Rightarrow$

$${}^{I}\phi_k^{\pm}(\mathrm{X}, \vec{W}) = \begin{cases} \tilde{\phi}_k(\vec{W}) e^{\mp i\omega(\mathrm{T} - \ln(\mathrm{X})/\kappa)} = \tilde{\phi}_k(\vec{W}) \left(\dfrac{x-t}{L} \right)^{\pm i\omega/\kappa} & \text{region I} \\ 0 & \text{region II} \end{cases}$$

Future-directed Killing vector $\xi = \partial_{\mathrm{T}}$

$${}^{II}\phi_k^{\pm}(\mathrm{X}) = \begin{cases} 0 & \text{region I} \\ \tilde{\phi}_k(\vec{W}) e^{\pm i\omega(\mathrm{T} - \ln(-\mathrm{X})/\kappa)} = \tilde{\phi}_k(\vec{W}) \left(\dfrac{t-x}{L} \right)^{\mp i\omega/\kappa} & \text{region II} \end{cases}$$

Future-directed Killing vector $\xi = \partial_{-\mathrm{T}} = -\partial_{\mathrm{T}}$

Note that ${}^{I}\phi_k^{\mp} = {}^{I}\phi_k^{\pm *}$ ${}^{II}\phi_k^{\mp} = {}^{II}\phi_k^{\pm *}$ and

$$ {}^{II}\phi_k^{-}(\mathrm{X},\vec{W}) \propto \left(\frac{t-x}{L}\right)^{i\omega/\kappa} = \left(\frac{-(x-t)}{L}\right)^{i\omega/\kappa} = \left(\frac{(x-t)}{L}\right)^{i\omega/\kappa} (e^{i\pi})^{i\omega/\kappa} = \left(\frac{(x-t)}{L}\right)^{i\omega/\kappa} e^{-\pi\omega/\kappa} $$

$$ \propto {}^{I}\phi_k^{+}(\mathrm{X},\vec{W}) $$

$\phi_k^p = {}^{I}\phi_k^{+} + {}^{II}\phi_k^{-}$ is a positive-frequency mode

Consider Minkowski vacuum $\mathbf{a}_{p,k}|0\rangle_M = 0$

$$ \mathbf{a}_{p,k} = \langle\langle \phi_k^p, \varphi \rangle\rangle = \langle\langle {}^{I}\phi_k^{+}, \varphi \rangle\rangle + \langle\langle {}^{II}\phi_k^{-}, \varphi \rangle\rangle = \mathbf{a}_{k,I} + e^{-\pi\omega/\kappa} \langle\langle {}^{II}\phi_k^{+*}, \varphi \rangle\rangle = \mathbf{a}_{k,I} - e^{-\pi\omega/\kappa}\, \mathbf{a}_{k,II}^{\dagger} $$

$$ \mathbf{a}_{p,k}|0\rangle_M = 0 \Rightarrow \mathbf{a}_{k,I}|0\rangle_M = e^{-\pi\omega/\kappa}\, \mathbf{a}_{k,II}^{\dagger}|0\rangle_M $$

correlations between modes in regions I and II

similarly

$\phi_k^{p'} = {}^{II}\phi_k^{+} + {}^{I}\phi_k^{-}$ is a positive-frequency mode

$$ \mathbf{a}_{p',k} = \mathbf{a}_{k,II} - e^{-\pi\omega/\kappa}\, \mathbf{a}_{k,I}^{\dagger} $$

$$ \mathbf{a}_{p,k}|0\rangle_M = 0 \Rightarrow \mathbf{a}_{k,II}|0\rangle_M = e^{-\pi\omega/\kappa}\, \mathbf{a}_{k,I}^{\dagger}|0\rangle_M $$

Minkowski vacuum $|0\rangle_M \propto \prod_k f(\mathbf{a}_{k,I}^{\dagger}, \mathbf{a}_{k,II}^{\dagger})\, |0\rangle_{\mathrm{I}}\, |0\rangle_{\mathrm{II}}$ Rindler vacuum

$$|0\rangle_M \propto \prod_k f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II}) |0\rangle_{\mathrm{I}} |0\rangle_{\mathrm{II}}$$

$$e^{-\pi\omega/\kappa}\ \mathbf{a}^\dagger_{k,II}|0\rangle_M = \mathbf{a}_{k,I}|0\rangle_M = \mathbf{a}_{k,I} f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II}) |0\rangle_{\mathrm{I}} |0\rangle_{\mathrm{II}} = \left[\mathbf{a}_{k,I}, f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II})\right] |0\rangle_{\mathrm{I}} |0\rangle_{\mathrm{II}}$$

$$\left.\begin{aligned} &e^{-\pi\omega/\kappa}\ \mathbf{a}^\dagger_{k,II} = \left[\mathbf{a}_{k,I}, f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II})\right] \\ \text{similarly}\quad &e^{-\pi\omega/\kappa}\ \mathbf{a}^\dagger_{k,I} = \left[\mathbf{a}_{k,II}, f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II})\right] \end{aligned}\right\} f(\mathbf{a}^\dagger_{k,I}, \mathbf{a}^\dagger_{k,II}) = \exp\left(e^{-\pi\omega/\kappa}\ \mathbf{a}^\dagger_{k,I}\mathbf{a}^\dagger_{k,II}\right)$$

Write $\mathbf{a}_{M,k} = \lambda \mathbf{a}_{p,k}$ and require $\delta^{(D-1)}\left(\vec{k} - \vec{k}'\right) = \left[\mathbf{a}_{Mk}, \mathbf{a}^\dagger_{Mk'}\right] = \lambda^2 \left[\mathbf{a}_{p,k}, \mathbf{a}^\dagger_{pk'}\right]$

$$\Rightarrow \delta^{(D-1)}\left(\vec{k} - \vec{k}'\right) = \lambda^2\left[\mathbf{a}_{k,I} - e^{-\pi\omega/\kappa}\ \mathbf{a}^\dagger_{k,II}, \mathbf{a}^\dagger_{k',I} - e^{-\pi\omega/\kappa}\ \mathbf{a}_{k',II}\right] = \lambda^2\left(1 - e^{-2\pi\omega/\kappa}\right)\delta^{(D-1)}\left(\vec{k} - \vec{k}'\right)$$

$$\mathbf{a}_{M,\omega} = \frac{1}{\sqrt{1 - e^{-2\pi\omega/\kappa}}}\left(a_{k,I} - e^{-\pi\omega/\kappa} a^\dagger_{k,II}\right) = \cosh r_\omega a_{k,I} - \sinh r_\omega a^\dagger_{k,II}$$

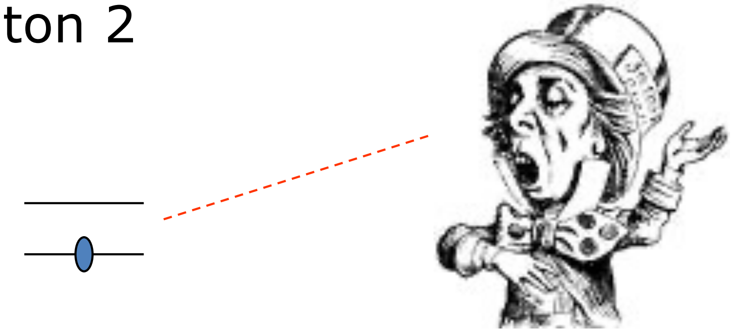
similarly

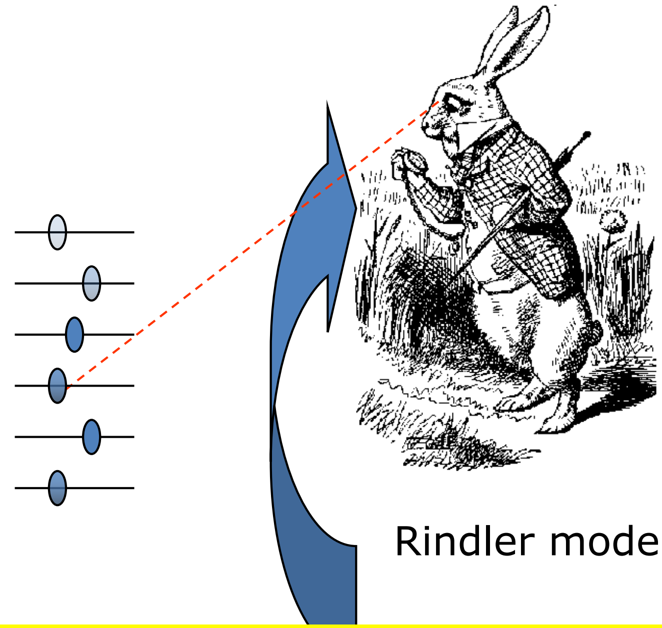
$$\mathbf{a}^\dagger_{M,\omega} = \frac{1}{\sqrt{1 - e^{-2\pi\omega/\kappa}}}\left(a_{k,II} - e^{-\pi\omega/\kappa} a^\dagger_{k,I}\right) = \cosh r_\omega a_{k,II} - \sinh r_\omega a^\dagger_{k,I}$$

$$\boxed{|0\rangle_M = \prod_\omega \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n(r_\omega) |n_\omega\rangle_{\mathrm{I}} |n_\omega\rangle_{\mathrm{II}}}$$

$\tanh r_\omega \equiv e^{-\pi\omega/\kappa}$

Photon 2





Rindler modes

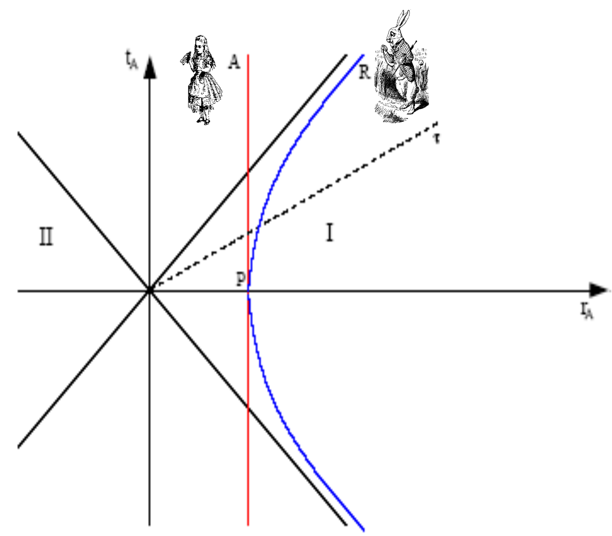
$$|0\rangle_2^B \Rightarrow |0\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r\,|n\rangle_I |n\rangle_{II}$$

$$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\Omega)}}$$

$$\Omega = \frac{\omega c}{a}$$

Rob is causally disconnected from region II

Unruh effect!



$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0_j\rangle_1^A |0_k\rangle_2^B + |1_j\rangle_1^A |1_k\rangle_2^B\right)$$

$$|0_k\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r\,|n_k\rangle_I |n_k\rangle_{II}$$

$$|1_k\rangle_2^B = \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}\tanh^n r\sqrt{n+1}\,|(n+1)_k\rangle_I |n_k\rangle_{II}$$

Trace over region II

$$\rho_{AR} = \mathrm{Tr}_{\mathrm{II}}\left[|\phi\rangle\langle\phi|\right] = \frac{1}{2\cosh^2 r}\sum_{n=0}^{\infty}\left(\tanh^{2n} r\right)\rho_n$$

$$|nm\rangle = |n_j\rangle^A |m_k\rangle_I$$

$$\rho_n = |0n\rangle\langle 0n| + \frac{\sqrt{n+1}}{\cosh r}|0n\rangle\langle 1(n+1)| + \frac{\sqrt{n+1}}{\cosh r}|1(n+1)\rangle\langle 0n| + \frac{n+1}{\cosh^2 r}|1(n+1)\rangle\langle 1(n+1)|$$

Entanglement?

Check eigenvalues of positive partial transpose

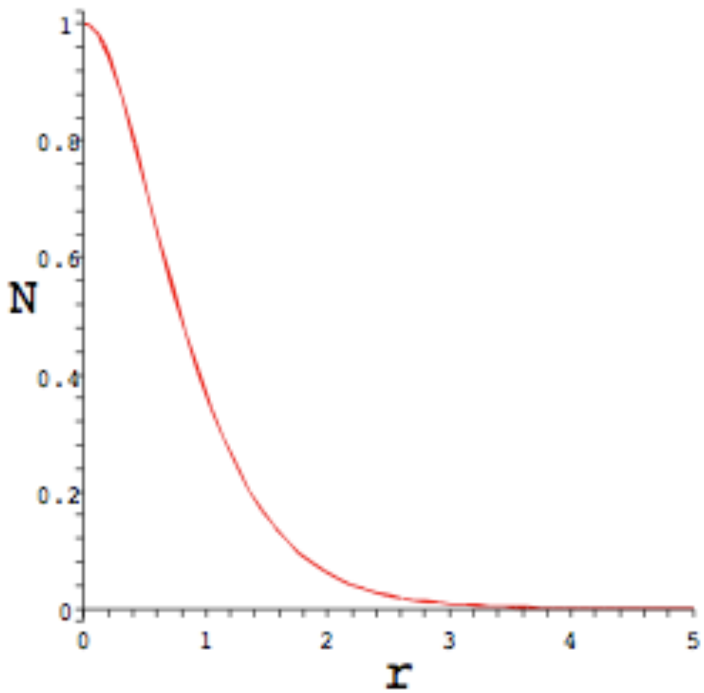
$$\lambda_{\pm}^{n} = \frac{\tanh^{2n} r}{4\cosh^2 r}\left[\left(\frac{n}{\sinh^2 r} + \tanh^2 r\right) \pm \sqrt{Z_n}\right]$$

$$Z_n = \left(\frac{n}{\sinh^2 r} + \tanh^2 r\right)^2 + \frac{4}{\cosh^2 r}$$

$$\lambda_{-}^{n} < 0$$

Always entangled for finite acceleration!

How much entanglement?



Logarithmic negativity vs r

$$N(\rho_{AR}) = \log_2\left(\frac{1}{2\cosh^2 r} + \Sigma\right)$$

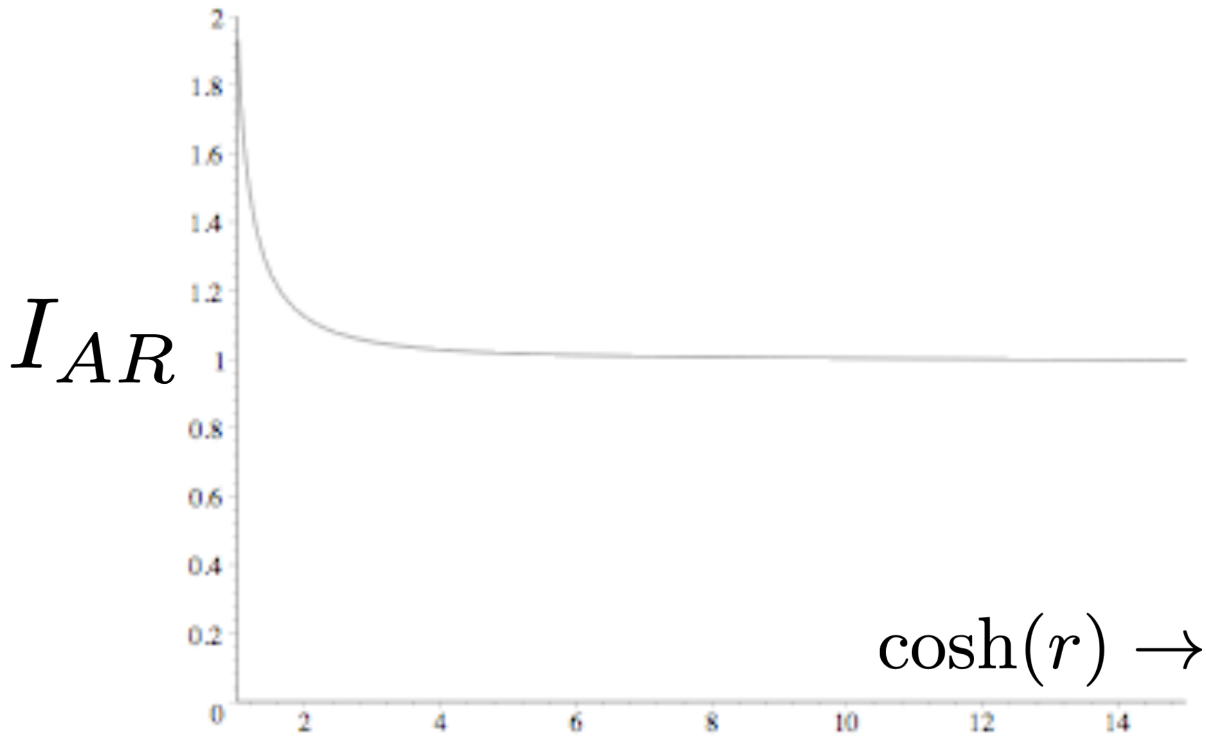
$$\Sigma = \sum_{n=0}^{\infty} \frac{\tanh^{2n} r}{2\cosh^4 r}\sqrt{\left(\frac{n}{\sinh^2 r} + \tanh^2 r\right)^2 + \frac{4}{\cosh^2 r}}$$

Distillable entanglement decreases as acceleration increases

Mutual Information

$$I_{AR} = S(\rho_A) + S(\rho_R) - S(\rho_{AR})$$

$$S(\rho) = -\mathrm{Tr}[\rho \log_2 \rho]$$



Mutual information vs cosh(r)

$$I_{AR} = 1 - \log_2(\tanh r) - \frac{1}{2\cosh^2 r} \sum_{n=0}^{\infty} \tanh^{2n} r D_n$$

$$D_n = \left(1 + \frac{n}{\sinh^2 r}\right) \log_2 \left(1 + \frac{n}{\sinh^2 r}\right)$$

In the infinite acceleration limit the total correlations consist of classical correlations plus bound entanglement

Where did the entanglement go?

Pure state: $S(\rho_{ARI,II}) = 0$ $\Rightarrow$ $S(\rho_{ARI}) = S(\rho_{RII})$

Entanglement between Alice+Rob in region I
With modes in region II

Zero acceleration

$$S(\rho_{ARI}) = 0$$

Alice+Bob are maximally entangled and there is no entanglement with region II

Finite acceleration

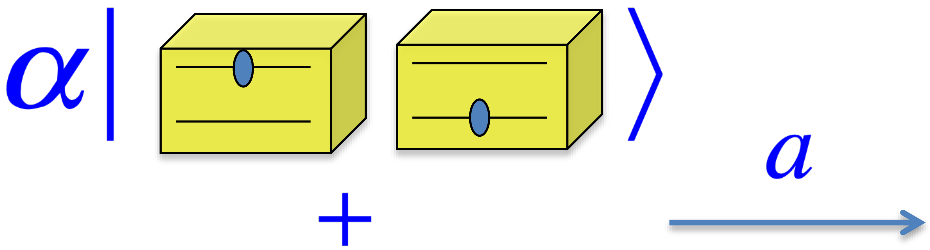
the entanglement between Alice+Rob is degraded and entanglement with region II grows

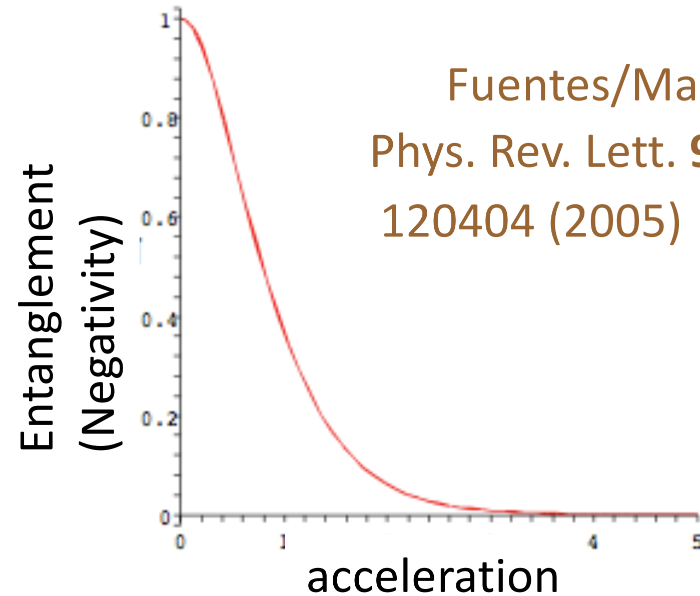
Infinite acceleration

$$S(\rho_{ARI}) = 1$$

Alice+Rob are disentangled and entanglement with region II is maximal

Acceleration Degrades Entanglement





Fuentes/Mann
Phys. Rev. Lett. **95** 120404 (2005)

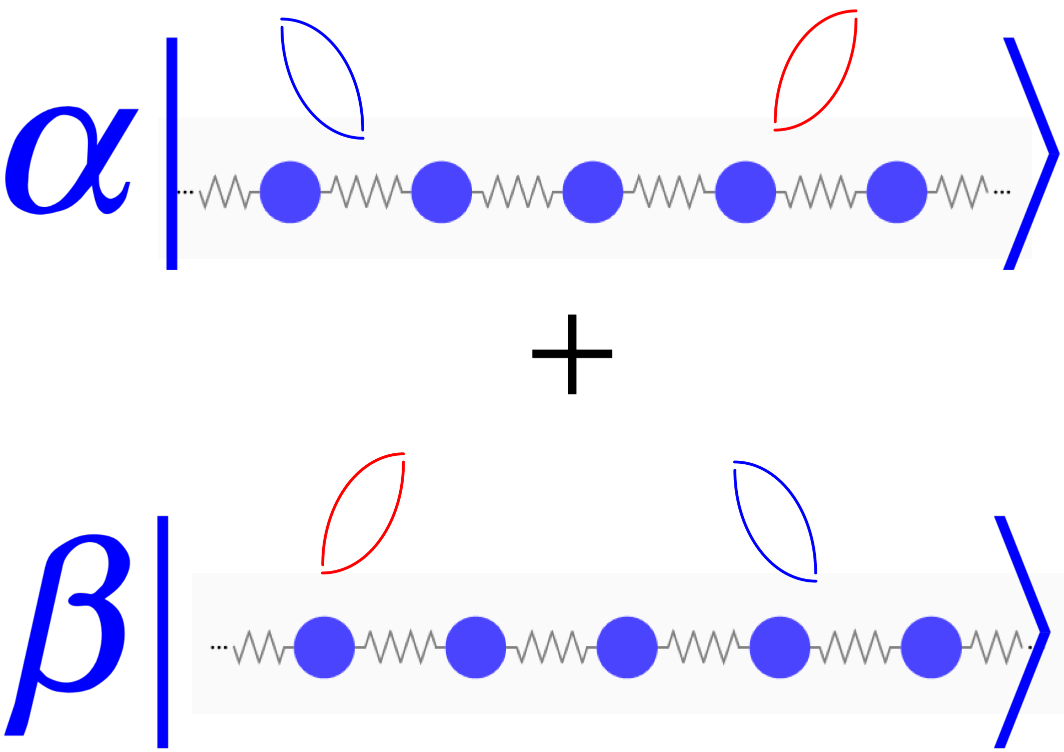
- 2 maximally entangled detectors lose entanglement if one of them accelerates!
- More acceleration → less entanglement!
- But gravity and acceleration are locally the same!
- → Gravity (curved spacetime) should affect quantum entanglement!

Alsing/Fuentes/Mann/Tessier
Phys. Rev. A**74** 032326 (2006)

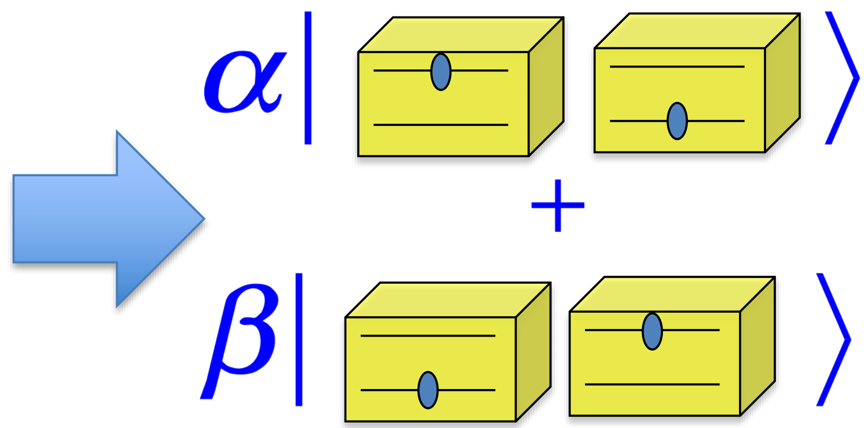
Entanglement Harvesting

- Extracting entanglement from the vacuum

Valentini
PLA **153**(1991) 321
Reznik Fnd Phy **33**
(2003) 167



Vacuum Entanglement



Detector Entanglement

Entanglement Harvesting

- Uncorrelated detectors in a vacuum can become entangled after a finite time
- Applications (in principle)
  - Seismology
  - Rangefinding
  - Quantum Key Distribution
  - Extraction from Atoms
  - Graphene

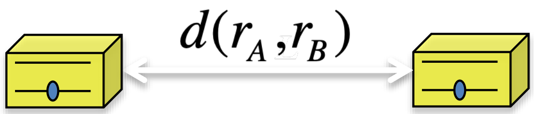
Brown/Donnelly/Kempf/RBM/
Martin-Martinez
NJP16 (2014)105020
Salton/RBM/Menicucci NJP17 (2015) 035001
Ralph/Walk NJP17 (2015) 063008
Pozas-Kerstjens /Martin-Martinez
PRD94 (2016) 064074; PRD95 (2017) 105009
Martin-Martinez/Sanders
NJP 18 (2016) 043031
Richter/Tercas/Omar/de Vega
PRA96 (2017) 053612
Huang/Tian NPB 923 (2017) 458
Rodríguez-Camargo/Svaiter/Menezes
Ann.Phys. 396 (2018) 266
Trevison/Yamaguchi/Hotta
PTEP10 (2018) 103A03
Ardenghi PRD98 (2018) 045006
Onoe/Ralph 1901.11144

Measuring Entanglement

$$\mathcal{C} = \max\{0, \underbrace{|X|}_{\text{Correlation}} - \underbrace{\sqrt{P_A P_B}}_{\text{Noise}}\}$$

Flat-Space Harvesting with Static Detectors

Pozas-Kerstjens/
Martin-Martinez
PRD92 (2015) 064042



Switching Width

$$\sigma_A = \sigma_B = T$$

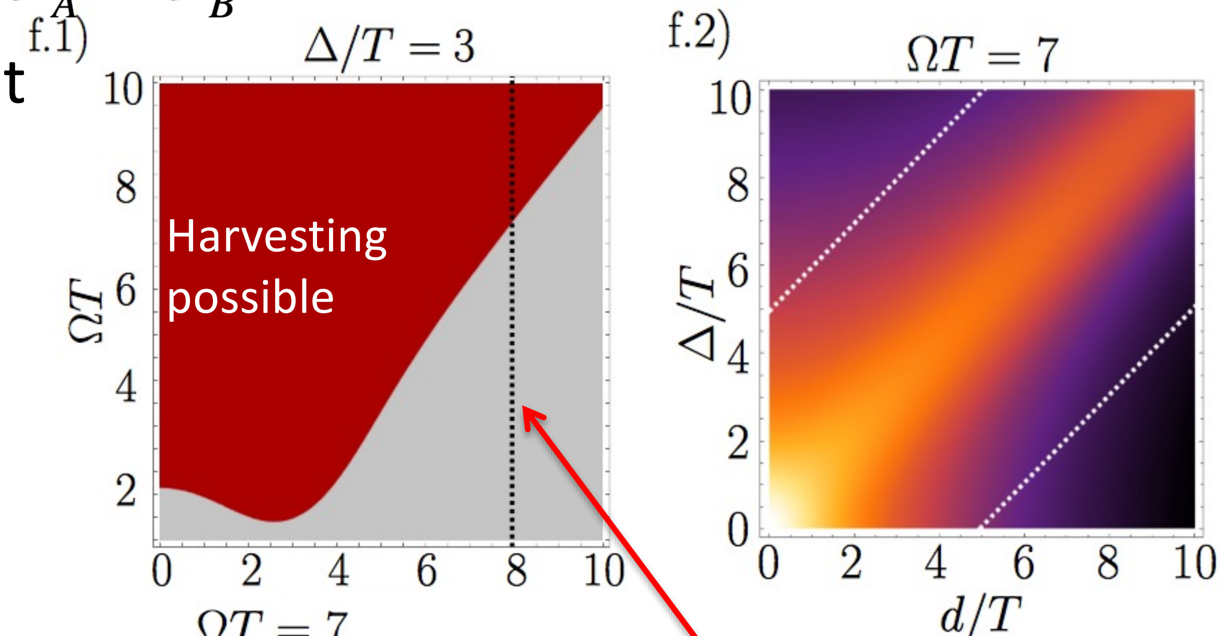
Switching Displacement

$$\tau_A - \tau_B = \Delta$$

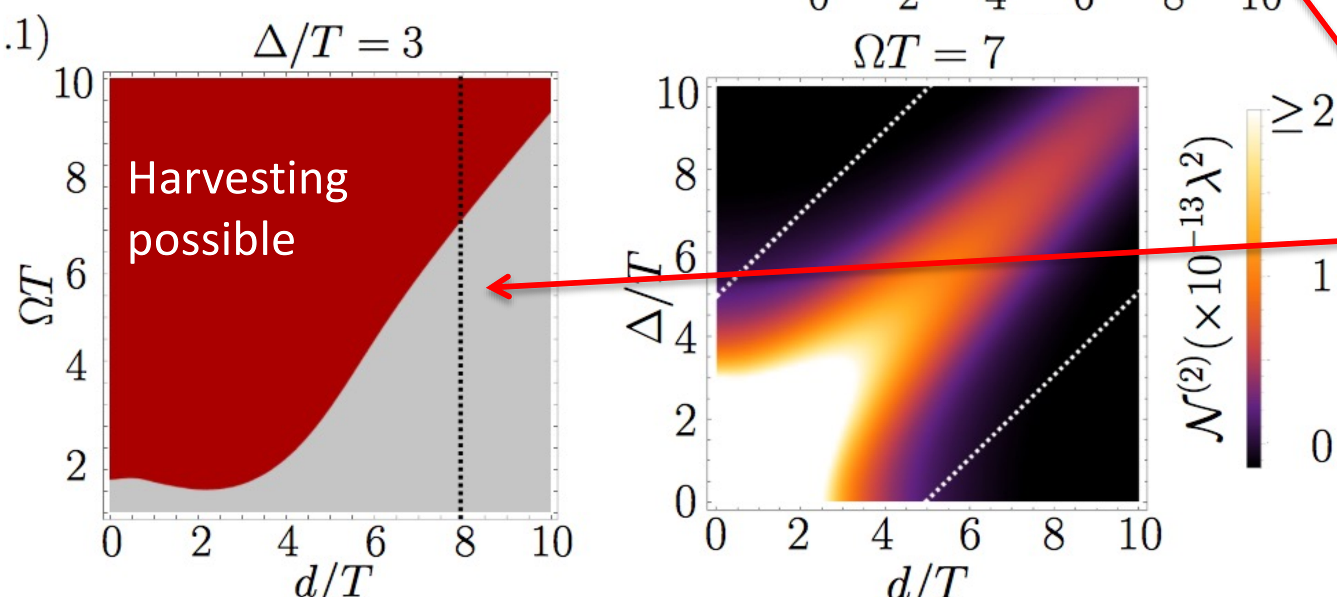
Detector Gap

$$\Omega_A = \Omega_B = \Omega$$

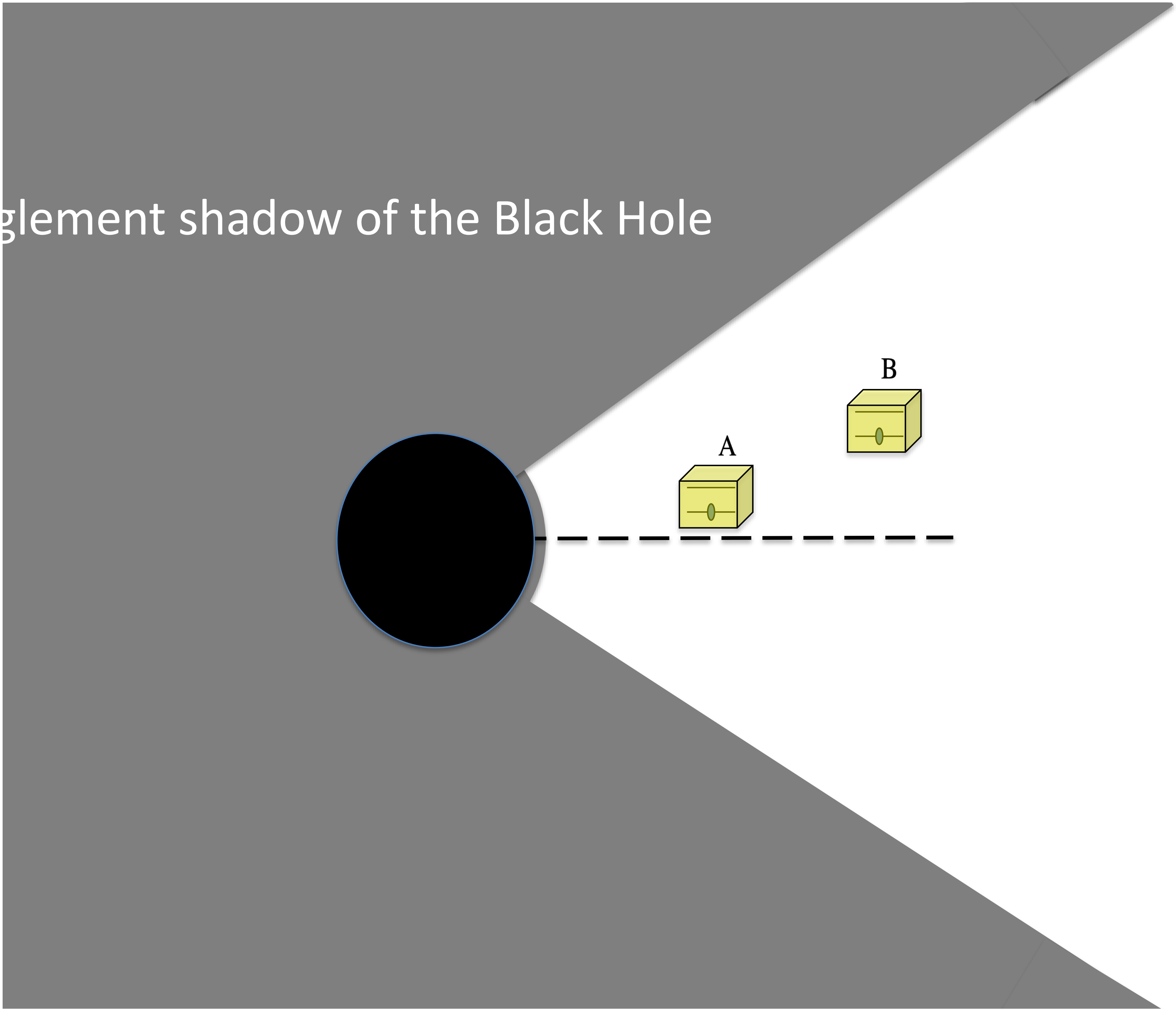
1+1



3+1



Spacelike separation


glement shadow of the Black Hole
B
A