Analog Regular Black Holes and Black Hole Mimickers in a Bose–Einstein Condensate

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Motivation

The simplest static, spherically symmetric **Regular Ultracompact objects (Regular UCOs)** cure the central singularity by promoting the ADM mass M in the metric functions to a radial mass function $m(r)$ controlled by a *regularization scale* ℓ, which determines the number of horizons. **Regular black holes (RBHs)** are regular UCOs with an inner and an outer trapping horizon. **Black hole mimickers (BHMs)** are horizonless UCOs that mimic black hole phenomenology without an outer horizon, but have as an additional feature a geometrically stable inner light ring.

Both are plagued by instabilities of purely **kinematic** origin:

- RBH: the inner (Cauchy) horizon suffers a blueshift/mass-inflation instability with growth rate $\sim c/|\kappa_-|$. If $|\kappa_-| \neq |\kappa_+|$, there is also a semiclassical instability.
- BHM: perturbations pile up in the potential minimum corresponding to the inner light ring, potentially leading to a nonlinear instability.

Since the origin is kinematic, both instabilities are testable at their *linear onset* in an analog platform, where we can test their robustness beyond standard relativistic dispersion. Our goal: realize the **entire** target UCO spacetime – not just an isolated horizon feature – in a Bose–Einstein condensate (BEC).

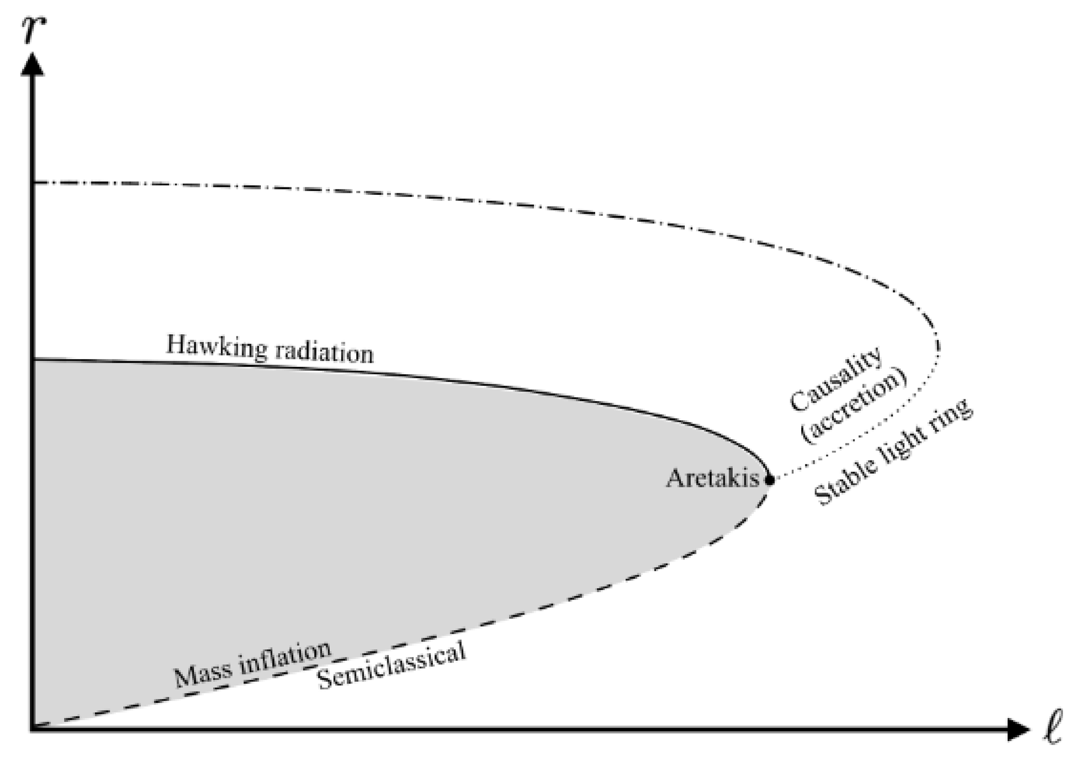

Fig. 2 – Schematic instability landscape of spherically symmetric UCO models as a function of the regularization scale ℓ (after Carballo-Rubio et al. 2025): mass inflation/semiclassical instabilities and Hawking radiation at the inner horizon in the RBH branch, and an accumulation (accretion) of perturbations at the stable inner light ring in the BHM branch.

Target Spacetime: Regular UCOs

Static, spherically symmetric family with mass function $m(r)$:

$$ds^2_{\rm gr} = -\left(1 - \tfrac{2m(r)}{r}\right)dt^2 + \left(1 - \tfrac{2m(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

e.g. Hayward model: $m(r) = \dfrac{Mr^3}{r^3 + 2M\ell^2}$.

All viable models share $m(r) \to M$ as $r \to \infty$ and $m(r) \to \text{const} \cdot r^3$ as $r \to 0$, giving a *finite* Kretschmann scalar $\mathcal{K} \to 24/\ell^4$ in the center and a regular de Sitter core.

ℓ controls the horizon structure, as well as the the horizon and light-ring locations:

- $\ell < \ell_* = 4M/\sqrt{27}$ (Hayward): two horizons $r_\pm \Rightarrow$ **RBH**.
- $\ell_* < \ell < \ell_{\rm deg}$: no horizons, two light rings $\Rightarrow$ **BHM**.

Sonic Metric ↔ Gravitational Metric

Linear perturbations of an irrotational BEC flow $\mathbf{v} = \nabla\phi$ obey a Klein–Gordon equation on the effective, 2+1D (dimensionally reduced, polar) *acoustic metric*

$$ds^2_{\rm ac} = -\left(c_s^2 - v_r^2\right)dt^2 - 2v_r\, dr\, dt + dr^2 + r^2 d\phi^2.$$

Rescaling time as $dt \to dt_{\rm ac} \equiv c_s(r)\, dt$ makes this line element **directly comparable** on the equatorial plane to the target gravitational one in Painlevé–Gullstrand form. This fixes the required flow through the following **correspondence relation**:

$$\boxed{\frac{v_r(r)}{c_s(r)} = -\sqrt{\frac{2m(r)}{r}}}$$

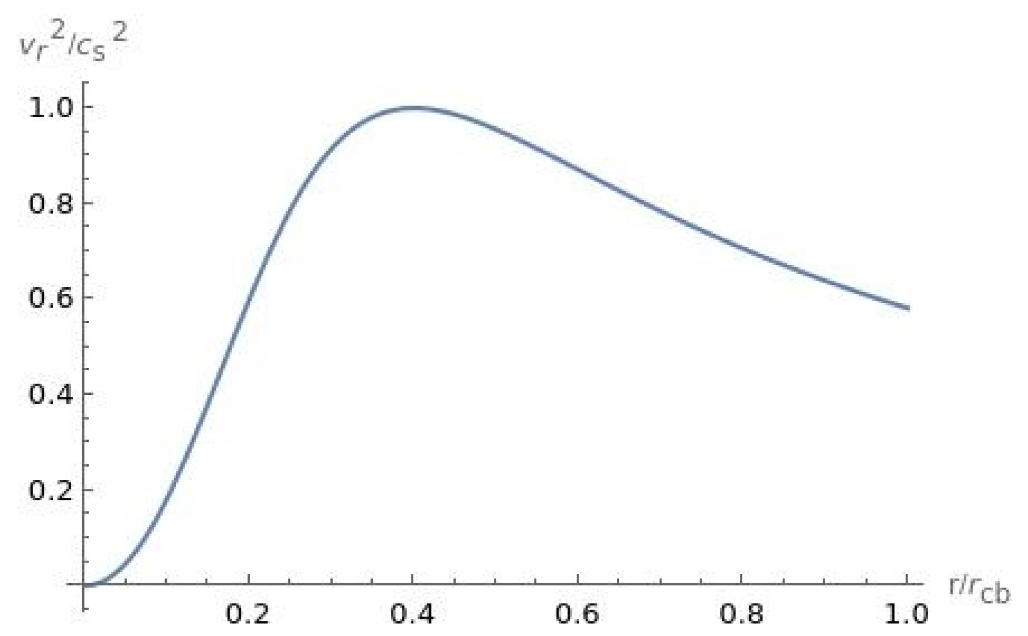

Fig. 1 – Required $(v_r/c_s)^2$ profile for a Hayward UCO. Peak < 1: horizonless BHM (shown); peak = 1: extremal RBH; peak > 1: RBH, with the two unity-crossings marking $r_\pm$. The object here is a horizonless BHM close to extremality.

Fluid Equations for the BEC Realization

In the Madelung representation, the Gross–Pitaevskii equation yields continuity and Bernoulli equations for a stationary, cylindrically symmetric flow:

$$\left(\rho v_r r\right)' = 0, \qquad \tfrac{1}{2}v_r^2 + U(r) + h(r) + Q(r) = \tfrac{\mu}{m_p},$$

with sound speed $c_s^2 = g(r)\rho(r)/m_p^2$, enthalpy $h(r) = c_s^2(r)$, and quantum pressure $Q(r) = \frac{\hbar^2}{2m_p^2}\frac{1}{\sqrt{\rho}}\frac{1}{r}\left(r\,\partial_r\sqrt{\rho}\right)$. Together with the **correspondence relation** above, this gives **3 equations** for the **4** variables (v_r, ρ, U, g). The external potential $U(r)$ *and* the interaction strength $g(r)$ – are experimentally tunable — unparalleled experimental flexibility.

Engineering $U(r)$ and $g(r)$

Core region ($r \to 0$, $m(r) \to r^3/2\ell^2$): for constant g, the quantum pressure *dominates* and completely erases any trace of ℓ — an obstruction. Requiring the enthalpy to beat quantum pressure fixes $g(r) \propto r^{-a}$, $a \geq 1$; for the most experimentally viable $a = 2$, one needs to engineer

$$U(r) \sim -\frac{2^{1/3} B g^{2/3}(r) l^{2/3}}{m_p^2 r^{4/3}} \propto -r^{-4/3} g^{2/3}(r) \propto -r^{-8/3}$$

and ℓ survives as a tunable constant in $U(r)$.

Asymptotic region ($r \to \infty$, $m(r) \to M$): the enthalpy term dominates automatically,

$$\left[U(r) - \frac{\mu}{m_p}\right] \to -\frac{B g^{2/3}(r)}{m_p^2 M^{1/3} r^{1/3}} \propto -r^{-1/3} g^{2/3}(r) \propto -r^{-5/3}$$

with M read off from the constant prefactor. Realized *globally*, this same behavior would yield an analog Schwarzschild black hole.

Feshbach engineering: $g(B) = \tilde{g}\left(1 - \frac{\Delta}{B - B_0}\right)$, so the required $g(r) = \tilde{g}\left(\frac{\tilde{r}}{r}\right)^2$ is produced by a simple two-coil magnetic trap $B(r) = B_0 + \Delta\big/\left(1 - (\tilde{r}/r)^2\right)$.

Proposed Experimental Platform

Quasi-2D BEC of ^{39}K ($m_p \approx 6.47 \times 10^{-26}$ kg), vertically confined by a tight dipole trap; $U(r)$ shaped via a digital micromirror device, $g(r)$ tuned via Feshbach coils. Using realistic parameters ($r_{\rm cb} = 40\,\mu$m, $r_{\rm in} \sim 1\,\mu$m, $n_{2D} = 2{\times}10^{13}\,$m^{-2}, $\mu/m_p = 3{\times}10^{-6}\,$J/kg, $\omega_r = 10^2\,$rad/s):

$$\ell \sim 10\ \mu\text{m}, \qquad M \sim 40\ \mu\text{m}.$$

Both are order-of-magnitude viable, and both scale steeply with experimental parameters – easy to tune, but also easy to fall outside the accessible window.

Instability Timescales & Observational Outlook

$$\tau_{\rm LR} \sim \frac{4M}{c_s} \approx 0.002\ \text{s} \quad \text{(inner light ring, BHM)}$$

$$\tau_{\rm IH} \sim \frac{c_s}{|\kappa_-|} \approx 0.01\ \text{s} \quad \text{(inner horizon, RBH)}$$

both comfortably within the ms-scale experimental time resolution $(\omega_z/2\pi)^{-1}$.

Expected signatures (dispersion $\omega^2 = c_s^2 k^2 (1 + 2k^2\xi^2)$, no backreaction): a **growing echo amplitude** tracing the inner light ring instability (BHM), and a **UV excess over the Hawking spectrum** concentrated near the frequency $|\kappa_-|/c_s$ (RBH) – made observable at infinity only because dispersive UV modes can escape the outer horizon.

Discussion & Outlook

- Analog gravity faithfully captures the linear, kinematic *onset* of both instabilities, but not the full nonlinear backreaction that decides the end state (collapse to a BH vs. dispersal to a less compact object).
- An unstable inner light ring would support the "black hole hypothesis" for EHT-imaged ultracompact objects; an unstable inner horizon suggests stationary Cauchy horizons are unphysical idealizations, motivating fully *dynamical* UCO formation scenarios.
- Proof-of-concept viable with present-day ^{39}K BEC technology; next steps include the experimental realization and the theoretical understanding of the relation of the black hole laser instability to the inner horizon instability with modified dispersion.

Reference & Contact

V. Pomakov, S. Liberati, *Analog regular black holes and black hole mimickers in a Bose-Einstein condensate* (will be on arXiv in the following days).
Companion construction for surface-gravity waves in shallow water: arXiv:2604.08671.

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