Characterisation of light rings around black holes in Scalar-Tensor-Vector Gravity

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1. Introduction

Circular photon orbits, commonly referred to as **light rings (LRs)**, are intimately **connected to several key observational signatures in black-hole (BH) physics** [1–6], and also provide valuable **information on their hypothetical mimickers** [7, 8]. Consequently, understanding their properties has become an **essential ingredient in the interpretation of strong-field observations and in testing gravity beyond General Relativity (GR)**. In this work, **we investigate the properties of LRs around BHs in Scalar-Tensor-Vector Gravity (STVG)**, also known as Modified Gravity (MOG) [9, 10], an extended theory capable of reproducing a variety of astrophysical phenomena without invoking particle dark matter. A particularly elegant and **general theoretical framework developed by Perlick** [11], based on the Hamiltonian formalism, **is here adopted** to describe light propagation through a collisionless, non-magnetised plasma surrounding the BH. Our analysis focuses on how the combined effects of the **modified spacetime geometry and the dispersive medium influence the existence, location and stability of LRs**. We also discuss the implications of our results for strong-field tests of gravity and future horizon-scale observations.

2. Potential-based approach for light rays

2.1 BACKGROUND BLACK-HOLE SPACETIME

We focus on a **stationary and axisymmetric BH solution in STVG** [10], reproducing Kerr metric in GR in the limit $\alpha \to 0$:

$$ds^2 = -\frac{\Delta}{\varrho^2}\left(dt - a\sin^2\vartheta\, d\varphi\right)^2 + \frac{\sin^2\vartheta}{\varrho^2}\left[(r^2+a^2)d\varphi - a\,dt\right]^2 + \frac{\varrho^2}{\Delta}dr^2 + \varrho^2 d\vartheta^2,$$

where $\Delta = r^2 - 2G_N(1+\alpha)Mr + a^2 + \alpha G_N^2(1+\alpha)M^2$ and $\varrho^2 = r^2 + a^2\cos^2\vartheta$.

In the "Kerr-like" metric above, ***M* and *a* are the BH mass and specific angular momentum**, respectively, and **α is the positive deviation parameter of STVG**, which quantifies the intensification in the gravitational coupling constant $G = (1+\alpha)G_{Newton}$.

A static, spherically symmetric "Schwarzschild-like" BH is obtained by setting $a = 0$.

2.2 HAMILTONIAN FORMALISM AND EFFECTIVE POTENTIALS IN PLASMA

Following Synge's prescription [12], **light rays** can be intended as **solutions to Hamilton's equations**:

$$\dot{x}^\mu = \frac{\partial\mathcal{H}}{\partial p_\mu}, \qquad \dot{p}_\mu = -\frac{\partial\mathcal{H}}{\partial x^\mu}.$$

In a **collisionless, non-magnetised test plasma**, sharing the **spacetime metric symmetries**, the Hamiltonian of radiation is

$$\mathcal{H} = \frac{1}{2}\left(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \omega_p^2\right) = 0,$$

where the plasma frequency takes the form

$$\omega_p(r,\vartheta) = \omega_c\sqrt{\frac{f_r(r)+f_\vartheta(\vartheta)}{r^2+a^2\cos^2\vartheta}}.$$

In this dispersive medium, **low-frequency light rays become timelike** with respect to the spacetime metric. At higher frequencies, the medium becomes transparent and null-geodesic behaviour in vacuum is recovered.

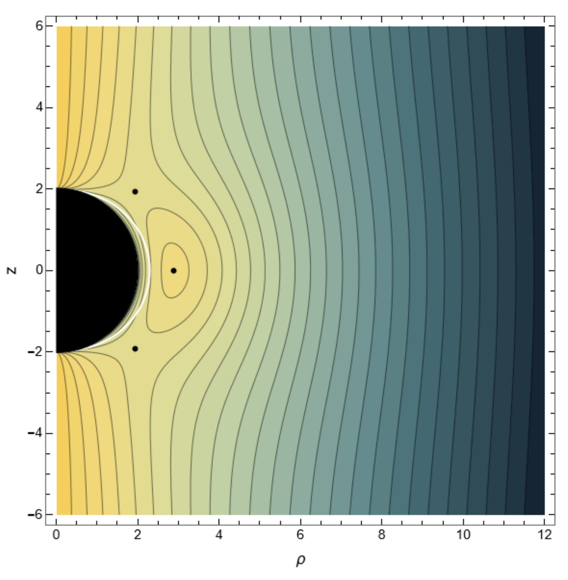

Figure 1: The potential $\mathcal{R}_+$ around a Kerr-like BH in STVG is plotted in the (ρ, z)-plane for a given choice of physical parameters and plasma frequency. Yellow colours indicate higher values of the potential. **The equatorial maximum and the saddle points correspond to three unstable LRs** around the BH, which is at the centre of the dark region, delimited by its event horizon and where the potential is not defined. The BH ergosphere is shown as a white line.

At points of light rays where $\dot{r} = \dot{\vartheta} = 0$, these **effective potentials** can be introduced [11]:

$$\mathcal{R}_\pm = \mp\frac{g_{t\varphi}}{g_{tt}}p_t + \frac{1}{g_{tt}}\sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}\sqrt{p_t^2 + \boldsymbol{\omega_p^2} g_{tt}}.$$

These potentials show gravity and **matter** influence on light propagation:

- **Gravitational lensing as the result of a "force"**, that is, their gradient;
- **LRs as their critical points**: stable LRs $\leftrightarrow$ local minima, unstable LRs $\leftrightarrow$ local maxima/saddles.

3. Exploring light-ring phenomenology

3.1 α-DEPENDENT DISPERSIVE ACTION OF PLASMA ON LIGHT RINGS

A plasma distribution efficiently enters the effective potentials $\mathcal{R}_\pm$ in the **low-frequency regime**:

$$\omega_0^2 \lesssim |g_{tt}|\omega_p^2.$$

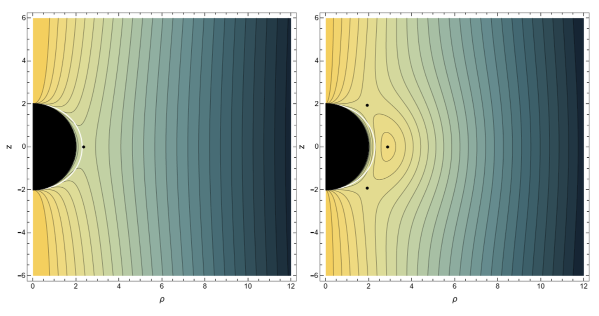

Figure 2: The left and right pictures show the potential $\mathcal{R}_+$ around a Kerr-like BH in vacuum and in plasma, respectively, for a given choice of physical parameters and plasma frequency. **The dispersive medium**, not negligible at the chosen frequency, modifies the equipotential lines and **leads to two non-equatorial LRs**.

What happens to LRs in plasma?

- **Low-frequency LRs form in multiple numbers** at larger distance from the BH;
- In the static case, LRs do not form a photon sphere anymore, when the plasma distribution is not spherically symmetric;
- LR orbital frequency lowers, consistent with being timelike trajectories in plasma;
- **LR instability is preserved**, even though their Lyapunov exponent is affected and, with it, their instability timescale.

A positive α alters the impact of plasma on LR formation, either enhancing or suppressing its dispersive action at a given frequency. This leads to:

- Variations in LR radius, orbital frequency and Lyapunov exponent;
- Possible **variation in LR number**;
- Unaltered LR instability: **BHs are stable ultracompact objects in both theories**.

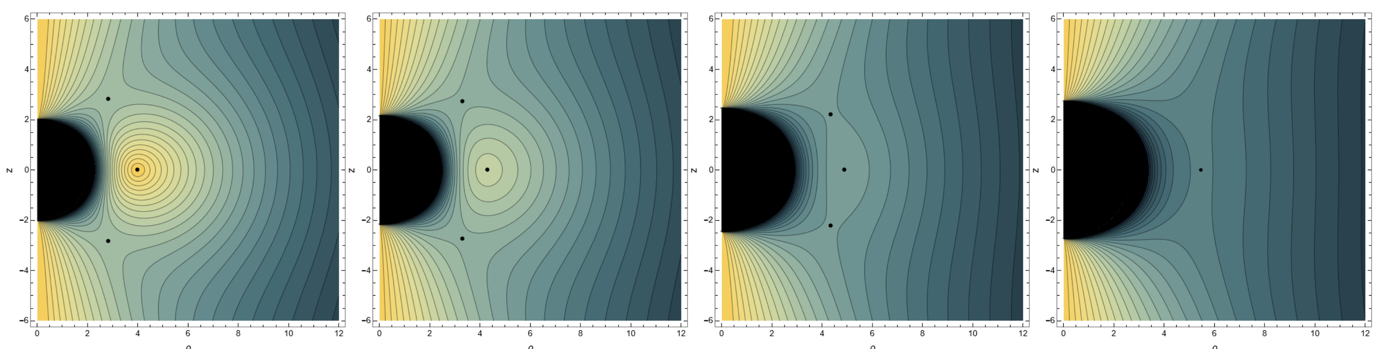

Figure 3: From left to right, the deviation parameter α is progressively increased from zero (GR) onwards in the case of a Schwarzschild-like BH in STVG, for a given choice of physical parameters and plasma frequency. The specific angular momentum is set to zero to isolate the impact of the deviation parameter on the dispersive action of the medium itself. In particular, even if the radiation frequency $p_t = -\omega_0$ is held constant, **as α is increased, LR distribution increasingly resembles the vacuum case: the medium becomes almost transparent to that frequency** in the end.

3.2 α-DEPENDENT IMPACT OF ROTATION ON LIGHT RINGS

How does BH spin affect LRs?

- **Prograde and retrograde LRs no longer coincide**;
- Even in vacuum, **LR spherical symmetry is broken** (no more photon spheres);
- **Plasma opacity to a given frequency is reduced**, possibly approaching the vacuum case.

A positive α reduces the impact of the BH spin on LR formation, making LR phenomenology closer to that around a static BH.

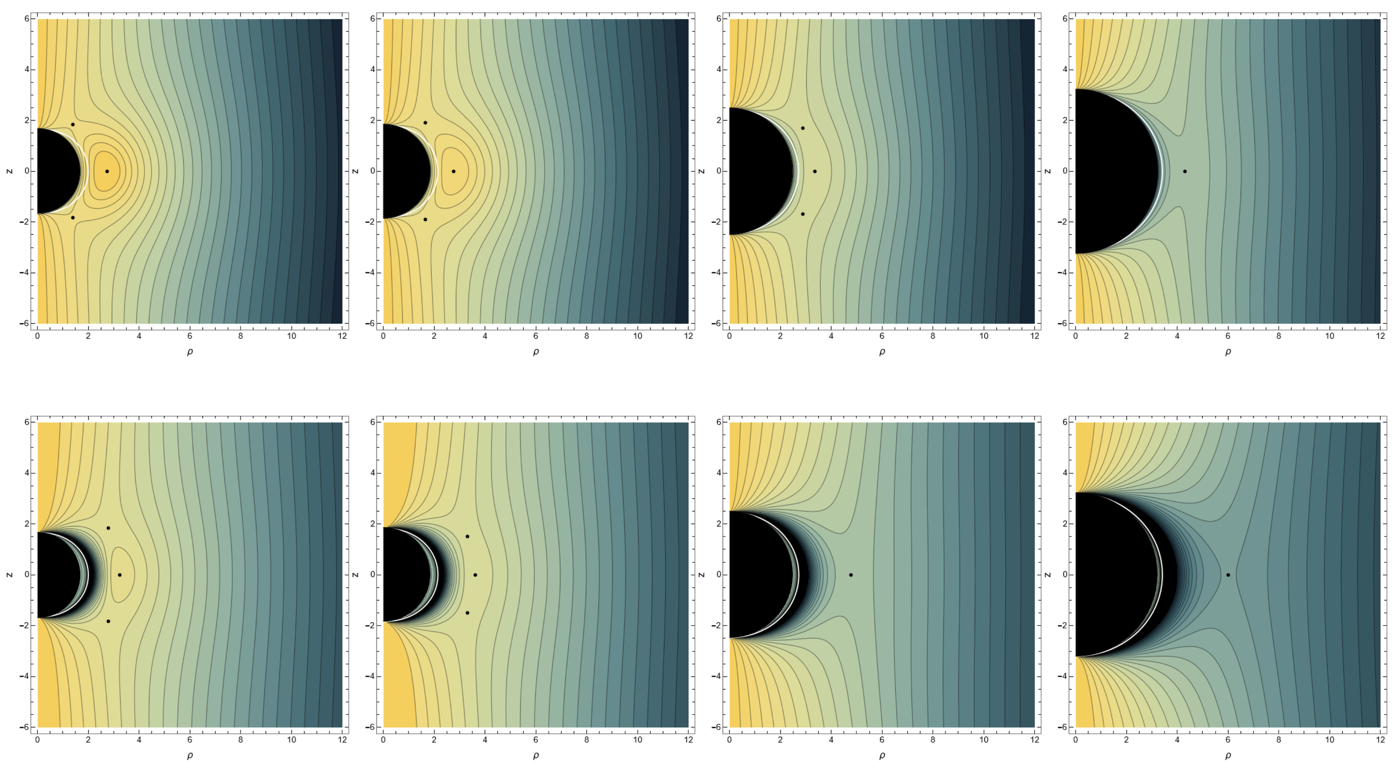

Figure 4: The upper and lower series show the evolution dictated by α of the potentials $\mathcal{R}_+$ and $\mathcal{R}_-$ around a Kerr-like BH in STVG, respectively, for a given choice of parameters and plasma frequency. Even if the BH specific angular momentum a is held constant, **as α is increased**, the event horizon and the ergosphere progressively enlarge, but the enclosed ergoregion thins out: **the two potentials become almost indistinguishable from that around a static BH**.

4. Observational implications and future perspectives

This work opens **a new line of investigation** into the strong-field regime of STVG, providing **the first characterisation of LRs around BHs** within this alternative framework.

Deviations from LR phenomenology in GR are purely geometrical effects, induced by the deviation parameter α, which impacts spacetime curvature and governs its interplay with the plasma in regulating LR formation.

The parameter α has a consequent impact at the level of BH images:

- **Larger shadow and surrounding ring-shaped emission**;
- **Less asymmetric shadow**, when the BH spin is misaligned with the observer's line of sight;
- **Different emission profile**, due to a colder accretion disc, orbiting the BH at a larger distance.

Adopting **realistic plasma models** and extending **ray-tracing codes** to STVG would yield simulations for **comparison with high-resolution data**, offering a way to **quantify STVG deviations from GR** or, at least, to place **the first constraints on α at the event-horizon scale**.

Introducing **other theoretical frameworks** and **BH mimickers** could be other future steps, further broadening the horizon of the investigation into the true nature of gravity.

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