Gravitational Signatures Beyond Newton: Exploring Hierarchical Three-Body Dynamics

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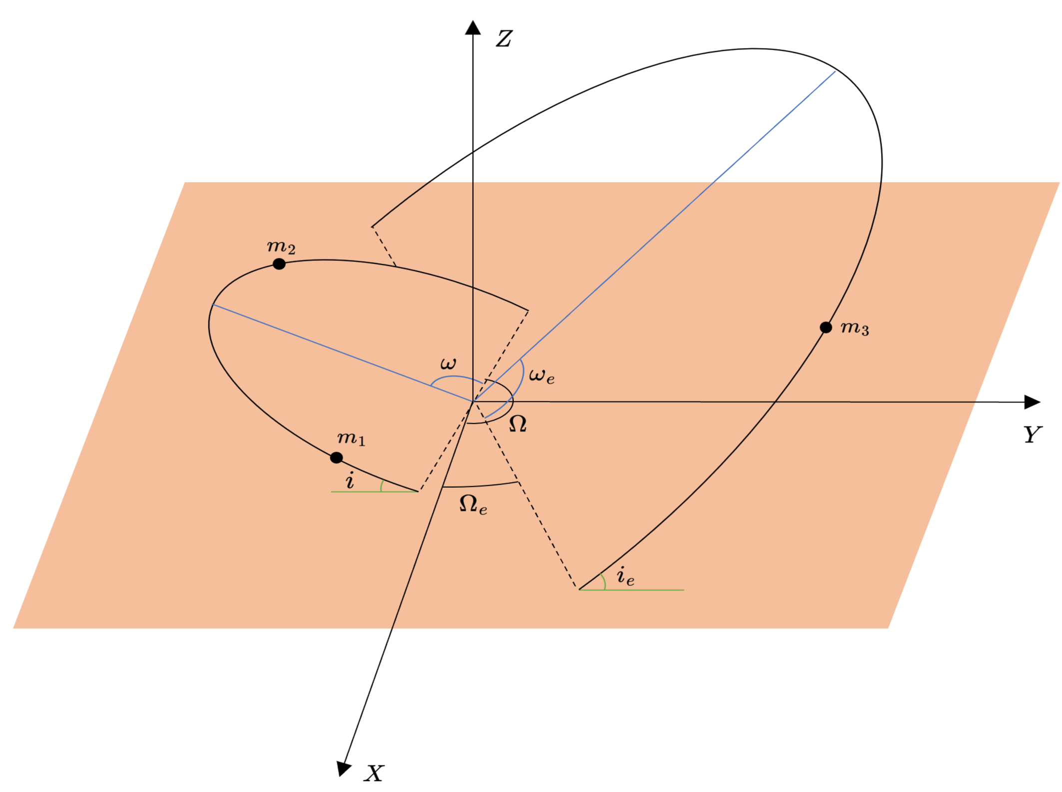
We investigate post-Newtonian corrections to the periastron shift within Hierarchical triplets, modeling the secondary body's influence as a quadrupolar perturbation. We use our results both for S87 star around Sagittarius A* and a hypothetical intermediate-mass black hole, and for small Solar System bodies. In addition, we present preliminary results based the Lagrangian approach, under more general assumptions.

First Method: Averaged Metric



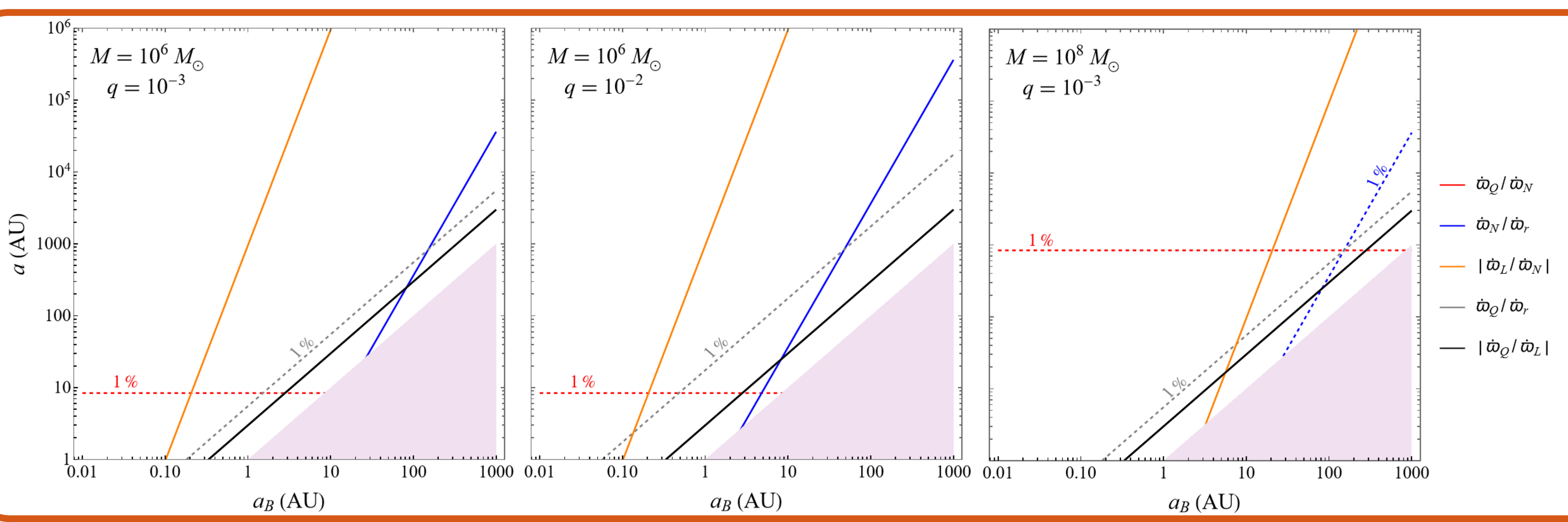
General geometry of a hierarchical triplet.

The averaged metric method is convenient to evaluate secular effects of the central binary to outer orbit (inverse ZKL). It is based on three fundamental steps *[K. Yamada, H. Asada (1962)]*:

1. We assume the following relation between masses: $m_1 \gg m_2 \gg m$ choosing a co-planar configuration.
2. Starting from the 1 PN metric generated by N point masses [L. Landau, E. Lifšitz (1962)], we expand $1/|\mathbf{r}-\mathbf{r}_a|$ up to quadrupole order.
3. We average over the binary period to obtain an effective metric and secular equations of motion for the outer body, from which we find:

$$\dot{\varpi}_Q = \frac{3Gm_2a_B^2}{8c^2a^3(1-e^2)^2}\left\{16+9e^2+e_B^2\left[24+\frac{e^4}{4}(65\cos 2\delta\omega+54)\right]\right\}n, \qquad \dot{\varpi}_L = -\frac{4Gm_2\sqrt{a_B(1-e_B^2)}}{c^2a^{3/2}(1-e^2)^{3/2}}n.$$

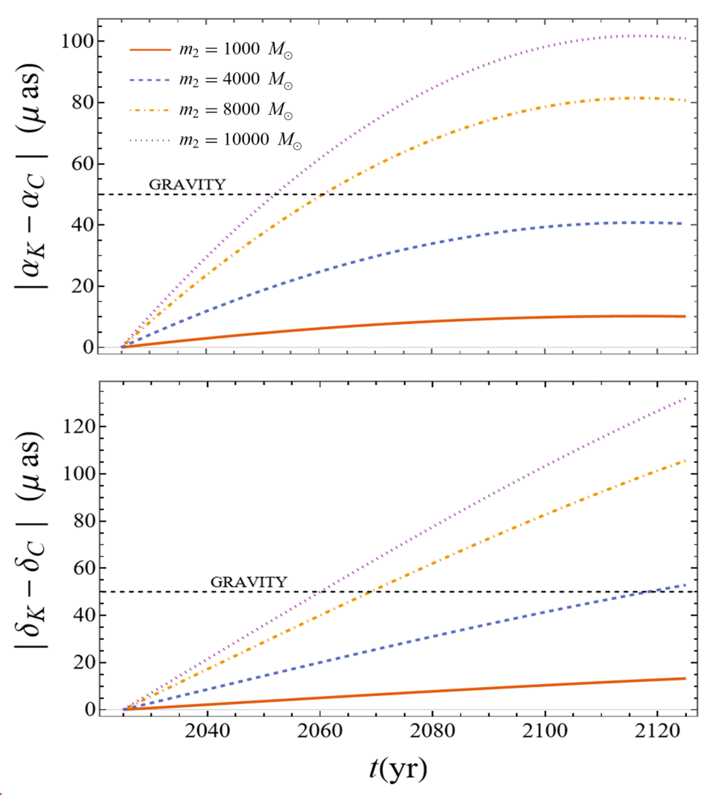
Comparisons



Ratios between different contributions. Solid lines indicate ratios equal to 1, dashed lines indicate ratios equal to 0.01. Above the lines the ratios become smaller than the respective value. The eccentricities are set to $e = 0.2$, $e_B = 0.5$.

PN quadrupolar perturbation represents almost less than 1% with respect to major contributions.
There are some parameter for which the angular momentum contribution exceeds the Newtonian one *(left panel)*.

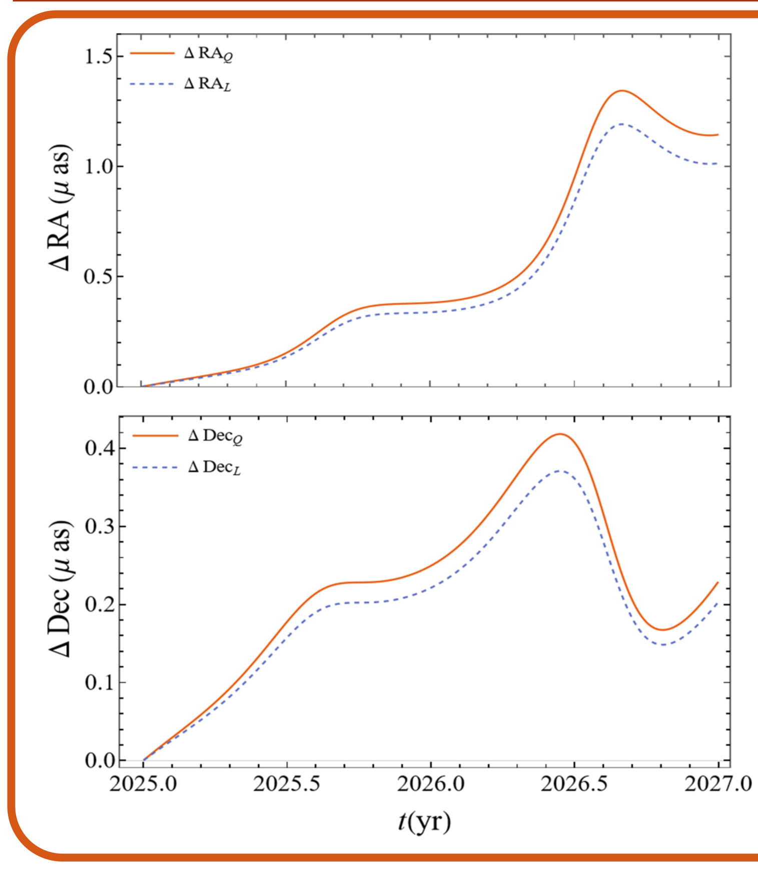
SgrA*-IMBH-S87



Difference in terms of celestial coordinates between a Keplerian orbit and one considering the Newtonian contribution, as a function of time. The parameters are $M = 4.3\times10^6 M_\odot$, $a_B = 7000$ AU, $e_B = 0.6$. The outer eccentricity is 0.224.

Newtonian effects on S87 could be observable, depending on the IMBH mass and/or the specific celestial coordinate, after about 25 to 95 yr.

Sun-Jupiter-Asteroid



Difference in terms of RA and Dec as a function of time between a Keplerian orbit and one where $\dot\varpi_Q$ and $\dot\varpi_L$ are considered. The outer semimajor axis is $a = 8$ AU The outer eccentricity is set to 0.1.

Post-Newtonian effects could be observable on an asteroid by Theia-like satellites since, in a timespan of 2 yr the difference in celestial coordinate can reach 1.4 μas.

Second Method: Perturbing Acceleration

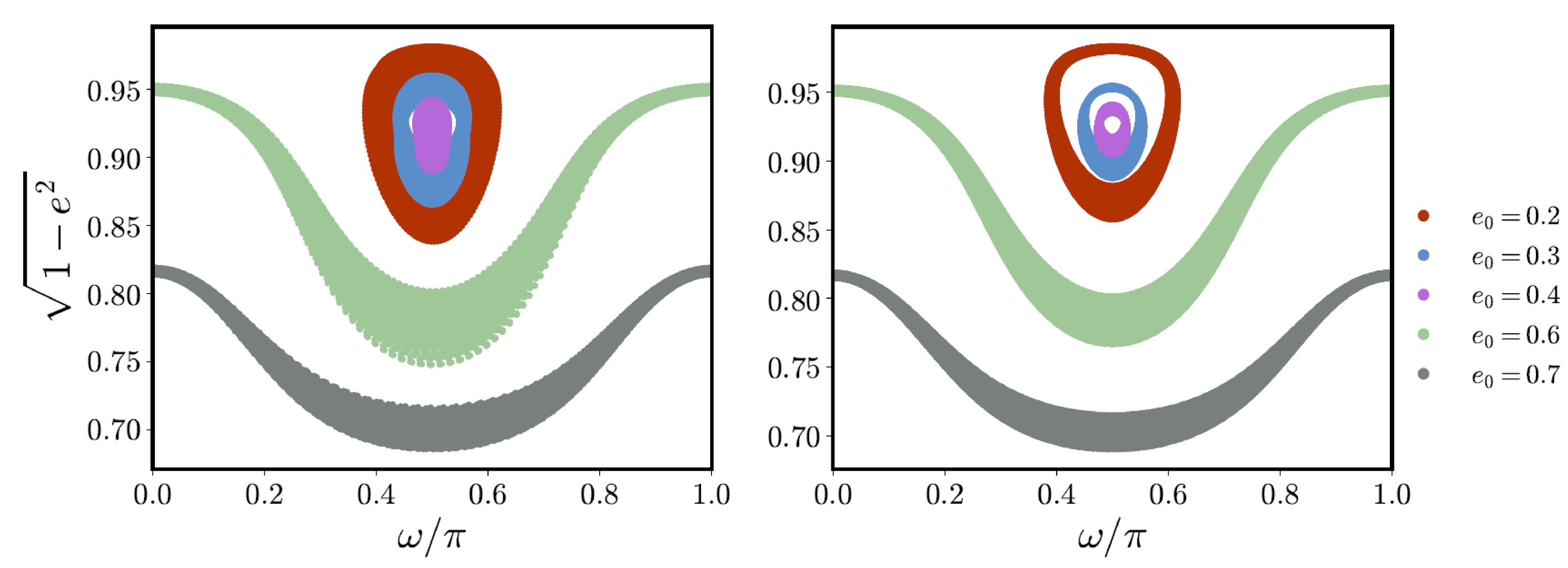
The Lagrangian method is mainly used to evaluate secular effect on the inner binary due to the presence of a massive perturber. It is based on the following steps:

1. Starting from EIH acceleration, we decompose the perturbation $\delta\mathbf{a} = \mathbf{a} + Gm\mathbf{n}/r^2$ on the three spatial directions, obtaining $\mathcal{R} = \delta\mathbf{a}\cdot\mathbf{n}$, $\mathcal{S} = \delta\mathbf{a}\cdot\lambda$, $\mathcal{W} = \delta\mathbf{a}\cdot\hat{\mathbf{h}}$.
2. Double-average the perturbed Lagrangian planetary equations *[C. M. Will, E. Poisson (2014)]*, e.g:

$$\frac{de}{dt} = \sqrt{\frac{p}{Gm}}\left(\mathcal{R}\sin\phi + \frac{2\cos\phi + e + e\cos^2\phi}{1+e\cos\phi}\mathcal{S}\right)$$

2.1. In some cases, it can be necessary include periodic corrections to double-average *[L. Luo et al. (2016)]*, throughout the ansatz:

$$X_\alpha(\theta,\tau;\phi_e) = \bar{X}_\alpha(\theta,\tau) + Y_\alpha(\bar{X}_\alpha(\theta,\tau),\phi_e)$$



Kozai cycles considering independent nodal precession (left panel) and assuming $\Delta\Omega = \pi$ (right panel) for different initial eccentricities. The parameters are $m_1 = 30\,M_\odot$, $m_2 = 20\,M_\odot$, $m_3 = 2\times10^7\,M_\odot$. The initial conditions are $e_3 = 0.8$, $a = 0.1$ AU, $a_3 = 210$ AU, $\omega = 90^\circ$, $\omega_3 = 280^\circ$, $\Omega = 193^\circ$, $\Omega_3 = 12^\circ$, $i = 135^\circ$.

For librating orbits, the span in of eccentricities reached during the cycle is larger than the case $\Delta\Omega = \pi$, while for circulating orbit this difference becomes smaller as the initial inner eccentricity increase.

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