Wormholes, ensembles, and non-invertible structures in 3D holography

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Euclidean wormholes pose a sharp puzzle for holography: while their bulk contributions naturally connect disconnected boundaries, an ordinary quantum field theory is expected to factorize across them. One possible resolution is that the gravitational theory is dual not to a single boundary theory, but to an ensemble of theories.
We explore this idea in a controlled setting based on three-dimensional "exotic" gravity theories, dual to ensembles of two-dimensional rational conformal field theories. In these models, it is possible to analyze multi-boundary amplitudes explicitly. Focusing on two disconnected boundaries, we ask whether "exotic wormhole" contributions are required by the consistency of the ensemble holography interpretation. It turns out that, in exotic gravity, these extra contributions have a precise bulk interpretation in terms of surface defects in a seed 3D TFT, which are known to form a non-invertible symmetry of the bulk theory.
We set up and study consistency conditions arising from ensemble holography with up to two boundary components in a number of exotic toy models. The emerging pattern is that consistency of the wormhole amplitude restricts the possible averaging over ensembles admitting an invertible or "group-like" multiplication structure.

Wormholes and non-factorization in 3D gravity

Pure AdS_3 gravity is related to 2D CFT with central charge $c = \frac{3\ell}{2G_N}$ (Brown and Henneaux, 1986; Witten, 2007)

Euclidean wormhole solutions are easy to construct (Maldacena and Maoz, 2004):

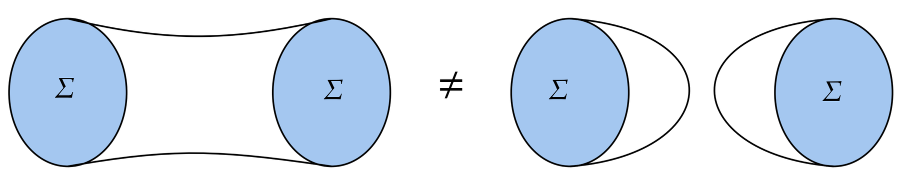
$$ds^2 = d\rho^2 + \cosh^2\rho \, ds^2_\Sigma, \qquad \rho \in (-\infty, \infty) \qquad \Rightarrow \partial\mathcal{M} = \Sigma \sqcup \Sigma$$

In the saddle-point approximation

$$Z_{\text{grav}}(\Sigma) = \sum_{\text{saddles}:\partial\mathcal{M}=\Sigma} e^{-S(g_\mathcal{M})}(1+\cdots)$$

they lead to a **non-factorization** of the partition function:

$$Z_{\text{grav}}(\Sigma \sqcup \Sigma) \neq Z_{\text{grav}}(\Sigma) Z_{\text{grav}}(\Sigma)$$



This contradicts the interpretation of AdS_3 gravity as a boundary CFT!

Motivated from 2D JT gravity (Saad, Shenker, and Stanford, 2019), generalized holographic proposal (Maloney and Witten, 2010; Cotler and Jensen, 2021; Jafferis, Rozenberg, and Wong, 2025):

Can pure AdS_3 gravity be dual to an ensemble average of 2D CFTs?

$$Z_{\text{grav}}(\Sigma) \stackrel{?}{=} \langle Z^{CFT}(\Sigma) \rangle$$

This would potentially resolve the non-factorization problem:

$$Z_{\text{conn}}(\Sigma_1, \Sigma_2) = \langle Z^{CFT}(\Sigma_1) Z^{CFT}(\Sigma_2) \rangle - \langle Z^{CFT}(\Sigma_1) \rangle \langle Z^{CFT}(\Sigma_2) \rangle \neq 0 \tag{1}$$

measures the **covariance** of the ensemble distribution. Connected n-boundary partition functions measure the higher moments.

Ensembles of boundary RCFTs

Toy models for AdS_3 gravity → restrict the boundary CFT ensemble to smaller families sharing an **extended symmetry algebra** and look for a bulk interpretation (Castro et al., 2012; Meruliya and Mukhi, 2021; Meruliya, Mukhi, and Singh, 2021).
↪ 2D CFTs with **rational** chiral algebra $\mathcal{A} \times \bar{\mathcal{A}}$.

Each RCFT is associated with a different **modular-invariant torus partition function**

$$Z_I(\tau) = \sum_{a,b} \overline{\chi_a(\tau)} M_I^{ab} \chi_b(\tau), \qquad \chi_a : \text{simple characters of } \mathcal{A} \tag{2}$$

$$M_I^{00} = 1, \qquad M_I^{ab} \in \mathbb{N}, \qquad [U_\gamma, M_I] = 0 \quad \forall \gamma \in SL(2,\mathbb{Z})$$

Ensemble averaging of $\mathcal{A}$-RCFTs, e.g. on $\Sigma = T^2$:

$$\langle Z^{CFT}(\tau) \rangle = \sum_I \rho^I Z_I(\tau), \qquad \text{ensemble weights:} \quad 0 \leq \rho^I \leq 1, \quad \sum_I \rho^I = 1.$$

Theory	c	Primaries	Modular-invariants
Compact boson @ $R^2 = k/\delta^2$	1	$a = 0, 1, \cdots, 2k-1$	divisors $\delta \vert k$
$\widehat{\mathfrak{su}}(2)_k$ WZW	$\frac{3k}{k+2}$	$a = 0, 1, \cdots, k$	A, D (k even), E ($k = 10, 16, 28$)
$\widehat{\mathfrak{su}}(3)_k$ WZW	$\frac{8k}{k+3}$	$(a_1, a_2) \in \mathbb{Z}^2_{\geq 0} : a_1 + a_2 \leq k$	A, C (charge conj.), D, D_C, E-type ($k = 5, 9, 21$)
$\widehat{\mathfrak{su}}(N)_1$ WZW	$N-1$	$a = 0, 1, \cdots, N-1$	divisors δ of $\begin{cases} N & N \text{ odd} \\ N/2 & N \text{ even} \end{cases}$

Often modular-invariant matrices close an **algebra** under matrix multiplication:

$$M_I M_J = \sum_K f_{IJ}^K M_K \tag{3}$$

Compact boson @ level k

$$M_\delta M_{\delta'} = M_{\gcd(\delta, \frac{k}{\delta}, \delta', \frac{k}{\delta'}) \frac{\delta\delta'}{\gcd(\delta,\delta')^2}}$$

$\widehat{\mathfrak{su}}(3)_k$, $3 \nmid k$

	A	C	D	D_C
A	A	C	D	D_C
C	C	A	D_C	D
D	D	D_C	A	C
D_C	D_C	D	C	A

$\widehat{\mathfrak{su}}(3)_k$, $3 \mid k$

	A	C	D	D_C
A	A	C	D	D_C
C	C	A	D_C	D
D	D	D_C	$3D$	$3D_C$
D_C	D_C	D	$3D_C$	$3D$

Bulk exotic gravity

Euclidean **exotic gravity** toy models from 3D TFTs based on the rational chiral algebra $\mathcal{A}$.

↪ Modular-invariant gravity partition function obtained from bulk **modular average** of the relevant **seed chiral TFT** (Maloney and Witten, 2020; Collier, Eberhardt, and Zhang, 2023):

$$Z_{\text{grav}}(\partial\mathcal{M}) \propto \sum_{\gamma \in \text{Map}(\partial\mathcal{M})/\text{Map}(\mathcal{M};\partial\mathcal{M})} |Z_{\text{TFT}}(\mathcal{M}^\gamma)|^2$$

Boundary RCFT	Bulk chiral TFT
Compact boson @ level k	$U(1)_k$ Chern-Simons
$\widehat{\mathfrak{su}}(N)_k$ WZW	$SU(N)_k$ Chern-Simons
Generic $\mathcal{A}$	Reshetikhin–Turaev TFT for $\mathcal{A}$

Ensemble holography with one T^2 boundary determines the ensemble weights,

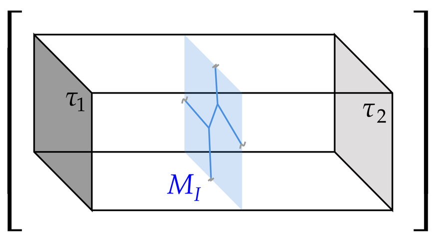
$$Z^{(1)}_{\text{grav}}(\tau) \stackrel{!}{=} \langle Z^{CFT}(\tau) \rangle \qquad \Leftrightarrow \qquad \rho^I. \tag{4}$$

Wormholes and surface defects

Once the ensemble weights ρ^I are fixed, so are all n-boundary connected correlators in the ensemble. On the bulk side, they should come from geometries connecting n boundary tori. With $n = 2$ this is the **wormhole amplitude**

$$\begin{aligned} Z^{(2)}_{\text{conn}}(\tau_1, \tau_2) &\propto \sum_\gamma |Z_{TFT}(T^2 \times I; \tau_1, \gamma \cdot \tau_2)|^2 \\ Z_{TFT}(\tau_1, \tau_2) &= \sum_I c^I Z_I(\tau_1, \tau_2), \qquad c^I : \text{wormhole coefficients} \end{aligned} \tag{5}$$

$Z_I(\tau_1, \tau_2)$ is the same modular-invariant appearing (2). In the bulk, it comes from a TFT wormhole amplitude in the presence of a **topological surface defect** ("exotic wormholes") (related to anyon condensation via "folding" (Benini, Copetti, and Di Pietro, 2023; Carqueville, Del Zotto, and Runkel, 2025))

$$Z_I(\tau_1, \tau_2) = Z_{TFT}\left[\vphantom{\int} \right.$$  $$\left. \vphantom{\int} \right]$$

Surface defects model the matrices M_I as bulk 0-form symmetry and **fuse** according to the (gen. non-invertible) rule (3) (Kapustin and Saulina, 2010; Roumpedakis, Seifnashri, and Shao, 2023).

Equating (5) and (1) determines the coefficients c^I and gives a set of highly nontrivial **constraints** on the ensemble weights ρ^I ⇒ Only certain ensembles are consistent!

Consistent ensembles from wormhole amplitudes

Solutions to the wormhole constraints can be studied using **representation theory** of the algebra spanned by the modular-invariant matrices M_I.

If the fusion rule (3) is **group-like** (as in (Raeymaekers and Rossi, 2024)):

1. $\rho^I = \delta^{I,\hat{I}}$ for a fixed modular-invariant $\hat{I}$. In this case there is **no ensemble average** and the gravity partition function factorizes, thus no Euclidean wormholes ($c^I = 0$).
2. ρ^I = const is the **uniform ensemble** distribution. In this case one finds $c^I \neq 0$, hence Euclidean wormholes must contribute to the connected partition function!

In many cases, (3) defines a **monoid** (**non-invertible**) multiplication structure. In all cases analyzed,

1. Nontrivial ensemble averaging is only possible within specific subgroups ⇒ $\rho^I \neq 0$ only for a subset of the M_I forming an **invertible** sub-symmetry.
2. Controlled, limiting exceptional cases exist.
3. Nontrivial ensemble averaging might be forbidden if the fusion rule (3) is too complicated.

→ Can invertible and non-invertible 0-form symmetries serve as an organizing principle for consistent CFT ensembles?
→ Extend ensemble holography consistency to multi-boundary partition functions?
→ What does this tell us about pure 3D gravity?

Based on

J. Raeymaekers, P.R., *Wormholes and surface defects in rational ensemble holography*, JHEP **01** (2024) 104, [arXiv:2312.02276]

P.R., work in progress