Higher-order nonlocal thermodynamic theories of gravity

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Abstract

By investigating Newtonian gravity within the framework of **nonequilibrium thermodynamics** it is possible to use specific energy density with up to 3rd order gradient dependency.

Assuming **cross-effect** between the mechanical and gravitational terms of the thermodynamic forces and fluxes arising from the specific entropy production rate, a generalised, and via the cross-effect, a **modified Poisson's equation** is obtained for the gravitational field.

The resulting nonrelativistic modified gravity may allow the explanation of various **astrophysical phenomena**. The method also naturally prevents the **Ostrogradsky-instability**, usually arising in higher-order theories.

Introduction

Adding higher order nonlocalities into a thermodynamic framework may be done via expanding the the specific internal energy with an appropriate general energy term:

$$u = e - \frac{\varepsilon(\rho, \varphi, \nabla\varphi, \nabla^2\varphi, \nabla^3\varphi)}{\rho},$$

where u is the specific internal energy, e is the specific total energy, ρ is the density, ε is a specific energy density dependent on the density, alongside of an arbitrary φ scalar field (this will be the nonrelativistic gravitational potential) and its $\nabla\varphi$, $\nabla\otimes\nabla\varphi = \nabla^2\varphi$ and $\nabla\otimes\nabla\otimes\nabla\varphi = \nabla^3\varphi$ gradients.

Simplified entropy balance

- Simpler case of $\varepsilon(\rho, \varphi, \nabla\varphi)$ for illustration
- Cross-effect between the gravitational and mechanical forces according to the Onsagerian relations; described by K
- The entropy balance from the II. law of thermodynamics:

$$\rho\dot{s} + \nabla\cdot\left[\frac{1}{T}(\mathbf{q} + \dot{\varphi}\partial_{\nabla\varphi}\varepsilon)\right] =$$
$$= \left(\mathbf{q} + \partial_{\nabla\varphi}\varepsilon\dot{\varphi}\right)\cdot\nabla\left(\frac{1}{T}\right) +$$
$$+\frac{\dot{\varphi}}{T}\left(\nabla\cdot(\partial_{\nabla\varphi}\varepsilon) - \partial_\varphi\varepsilon\right) -$$
$$-\left[\mathbf{P} + (\varepsilon - p - \rho\partial_\rho\varepsilon)\mathbf{I} - (\partial_{\nabla\varphi}\varepsilon)(\nabla\varphi)\right] : \frac{\nabla\mathbf{v}}{T} \geq 0$$

Termodynamic gravity

- According to Ván & Abe, the energy density of the gravitational field has the following distinguished form [1, 2]:

$$\varepsilon_{TG} = \varphi\rho + \frac{(\nabla\varphi)^2}{8\pi G}$$

- In a stationary case, the following field equation is obtained:

$$\boxed{\Delta\varphi = 4\pi G\rho + K(\nabla\varphi)^2}$$

- Its spherically symmetric vacuum solution:

$$g(r) = -\frac{GM}{KGMr + r^2},$$
$$\varphi(r) = \frac{1}{K}\ln\frac{1}{KGM/r + 1}$$

- The perihelion precession of a planetary orbit ($KGM \ll a$):

$$\Delta\phi = -2\pi\frac{KGM}{2a(1-\varepsilon^2)}$$

Yukawa-type gravity

- The extension investigated by Lazar [3], within the thermodynamics framework is the following case:

$$\varepsilon_{Yukawa} = \frac{1}{8\pi G}\Big((\nabla\varphi)^2$$
$$+ l_1^2\left(\nabla^2\varphi\right)^2 + l_2^4\left(\nabla^3\varphi\right)^2\Big) + \rho\varphi$$

- The stationary field equation:

$$\boxed{\begin{aligned} 4\pi G\rho &= \Delta\varphi - l_1^2\Delta^2\varphi + l_2^4\Delta^3\varphi - \\ -K\Big((\nabla\varphi)^2 &+ l_1^2\left[\left(\nabla^2\varphi\right)^2 + \Delta(\nabla\varphi)^2\right] + \\ +l_2^4&\left[\left(\nabla^3\varphi\right)^2 + \Delta(\nabla^2\varphi)^2 - \Delta^2(\nabla\varphi)^2\right]\Big) \end{aligned}}$$

- The vacuum solution with $l_2 = 0$, $K = 0$, typically used in the description of equivalence principle-breaking modifications:

$$\varphi(r) = \frac{GM}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Self-consistent, self-coupling gravitation

- The mass of the gravitational field's energy (in the special relativistic sense) can be considered within density distribution responsible for the field itself.
- The self-consistent solution for this approach has been investigated by many [4], and corresponds to this case:

$$\varepsilon_{self-cons.} = \frac{c^2}{8\pi G\varphi}(\nabla\varphi)^2 + \rho\varphi$$

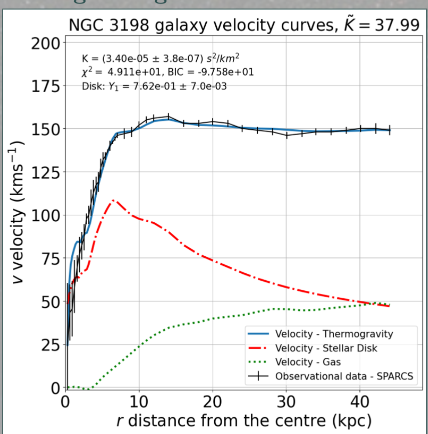
- In the stationary case, the following field equation is obtained:

$$\boxed{\Delta\varphi = \frac{1}{2}\left(\frac{1}{\varphi} + 2K\right)(\nabla\varphi)^2 + \frac{4\pi G}{c^2}\rho\varphi}$$

- Its analytical vacuum solution:

$$\varphi(r) = \frac{1}{K}\mathrm{erf}^{-1}\left(\sqrt{\frac{K}{c^2\pi}}\left(2c^2 - \frac{GM}{r}\right)\right)^2$$

Testing with galactic rotation curves



Using the observational data from the SPARC [5] database allows us to test a gravitational theory. The joint contribution of the stellar disk and the gas is unable to explain the observed **rotational velocities**, by only using Newtonian dynamics/gravity. The thermogravity model can be numerically solved through the relaxing equation using the density data, thus fitting the velocity curve without invoking dark matter [6, 7, 8]. → QR-code

The Eötvös torsion balance. According to experiments, $\lambda = l_1 \lesssim 4 \times 10^{-5}$ m [9].

Theory

Numerical solution

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The work was supported by the Advanced 150038 grant of the National Research, Development and Innovation Office.