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Towards a causal effective thermodynamics of scalar-tensor gravity [1]

Core problem: Mapping scalar-tensor gravity into Eckart's first-order thermodynamics means formally connecting gravity to a theory that intrinsically suffers from acausal instabilities.

Our solution: We cure this pathology by reformulating the formal map to the **Israel-Stewart second-order causal theory**, validating the framework on exact cosmological solutions.

1. Effective Fluid Interpretation

- **Effective dissipative EMT:** Mapping classes of scalar-tensor gravity into Eckart's thermodynamics yields:

$$\mathcal{G}_{ab} = \frac{8\pi}{\phi} T^{(m)}_{ab} + \underbrace{\frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}(\nabla_a\nabla_b\phi - g_{ab}\Box\phi) - \frac{V}{2\phi}g_{ab}}_{\equiv 8\pi T^{(\phi)}_{ab} \rightsquigarrow \text{Effective dissipative } \phi\text{-fluid EMT}} . \quad (1)$$

- **Fluid decomposition:** Defining the purely kinematic 4-velocity as

$$u^a \equiv \frac{\nabla^a\phi}{\sqrt{-\nabla_a\phi\nabla^a\phi}} ,$$

$T^{(\phi)}_{ab}$ decomposes into standard effective (ϕ-)fluid variables ($\rho^{(\phi)}$, $P^{(\phi)}$, $q^{(\phi)}_a$, $\pi^{(\phi)}_{ab}$).

- **Effective Temperature:** Applying Eckart's Fourier Law, we obtain the thermal parameters in the Jordan Frame [3]:

$$\mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla_a\phi\nabla^a\phi}}{8\pi\phi} \quad (2)$$

Key property: $\mathcal{K}\mathcal{T}$ works as the **order parameter** for the relaxation process toward the stable equilibrium state. General Relativity ($\phi = \text{const.}$) is naturally recovered when $\mathcal{K}\mathcal{T} \to 0$.

2. The Pathology: Acausality

Eckart's constitutive equation for the heat flux [2]

$$q_a = -\mathcal{K}(h_{ab}\nabla^b\mathcal{T} + \mathcal{T}\dot{u}_a) \quad (3)$$

is mathematically parabolic, allowing infinite propagation speeds.

Eckart's Theory	→	Israel-Stewart
1st Order PDE		2nd Order PDE
Parabolic system		Hyperbolic system (τ_i)
× *Acausal Instabilities*		✓ *Causality Restored*

3. Israel-Stewart Constitutive Equations

We fix the acausality by upgrading to the Israel-Stewart transient model [4]:

$$\tau_0\,\dot{\Pi} + \Pi = -\zeta\Theta - \frac{\zeta\mathcal{T}}{2}\left[\nabla_c\left(\frac{\tau_0 u^c}{\zeta\mathcal{T}}\right)\right]\Pi \quad (4)$$

$$\tau_1\, h_a{}^b\dot{q}_b + q_a = -\mathcal{K}\left(h_{ab}\nabla^b\mathcal{T} + \mathcal{T}\dot{u}_a\right) - \frac{\mathcal{K}\mathcal{T}^2}{2}\left[\nabla_b\left(\frac{\tau_1 u^b}{\mathcal{K}\mathcal{T}^2}\right)\right] q_a \quad (5)$$

$$\tau_2\, h_a{}^c h_b{}^d\dot{\pi}_{cd} + \pi_{ab} = -2\eta\,\sigma_{ab} - \frac{\eta\mathcal{T}}{2}\left[\nabla_c\left(\frac{\tau_2 u^c}{\eta\mathcal{T}}\right)\right]\pi_{ab} \quad (6)$$

Minimal ansatz: Assuming the heat flux q^a is time-like ($q^a q_a < 0$):

$$q_a = (-q_c u^c)u_a + h_a{}^b q_b \equiv (q^{(\parallel)}, q^{(\perp)}) \qquad \text{with} \quad h_a{}^b q_b^{(\parallel)} = 0\,, \quad q_a^{(\perp)}u^a = 0\,.$$

4. Causal Thermodynamics

Applying the causal minimal ansatz and restricting to the subsystem where the generalized Eckart's laws hold, we obtain:

- **Generalized heat flux:**

$$q_a = -\mathcal{K}(\nabla_a\mathcal{T} + \mathcal{T}\dot{u}_a) \quad (7)$$

- **Strictly positive viscous energy density:**

$$\rho_v = -2q_c u^c = 2\mathcal{K}\dot{\mathcal{T}} > 0 \quad \Longrightarrow \quad \rho = \rho_{\rm pf} + \rho_v \quad (8)$$

- **Invariant $\mathcal{K}\mathcal{T}$ and relaxation to GR ($\mathcal{K}\mathcal{T} \to 0$):**

$$\frac{\mathrm{d}(\mathcal{K}\mathcal{T})}{\mathrm{d}\tau} = 8\pi(\mathcal{K}\mathcal{T})^2 - \Theta\mathcal{K}\mathcal{T} + \frac{\Box\phi}{8\pi\phi} \quad (9)$$

- **Reduced Israel-Stewart PDEs:**

$$\tau_0\,\dot{\Pi} + \frac{\zeta\mathcal{T}}{2}\left[\nabla_c\left(\frac{\tau_0 u^c}{\zeta\mathcal{T}}\right)\right]\Pi = 0 \quad (10)$$

$$\tau_1\, h_a{}^b\dot{q}_b + \frac{\mathcal{K}\mathcal{T}^2}{2}\left[\nabla_b\left(\frac{\tau_1 u^b}{\mathcal{K}\mathcal{T}^2}\right)\right] q_a - (q_c u^c)u_a = 0 \quad (11)$$

$$\tau_2\, h_a{}^c h_b{}^d\dot{\pi}_{cd} + \frac{\eta\mathcal{T}}{2}\left[\nabla_c\left(\frac{\tau_2 u^c}{\eta\mathcal{T}}\right)\right]\pi_{ab} = 0 \quad (12)$$

5. Application to Cosmology

We test the mapping on exact backgrounds to check the thermal variables.

Bianchi I Cosmology

$$\mathrm{d}s^2 = \mathrm{d}t^2 + a_1^2(t)\mathrm{d}x^2 + a_2^2(t)\mathrm{d}y^2 + a_3^2(t)\mathrm{d}z^2\,, \qquad a(t) \equiv [a_1(t)a_2(t)a_3(t)]^{1/3}\,.$$

This anisotropic geometry allows for a remarkable mathematical achievement: the **first consistent decoupling** of the effective thermal conductivity and temperature:

$$\mathcal{K} = \frac{\mathcal{K}_0}{\tau_0 a\dot{a}^2}\,, \qquad \mathcal{T} = \frac{\tau_0 a\dot{a}^2}{8\pi\mathcal{K}_0}\frac{|\dot{\phi}|}{\phi}\,. \quad (13)$$

FLRW Cosmology

In the isotropic limit ($a_1(t) = a_2(t) = a_3(t)$), the viscous components mimic a radiation equation of state, $P_v/\rho_v = 1/3$. The relaxation time τ_0 evolves according to:

$$\tau_0 = \frac{C_0 a^{7/2}\phi}{\dot{a}^2}\,, \qquad \frac{\dot{\tau}_0}{\tau_0} = \frac{\Theta}{2} - \frac{2\dot{\Theta}}{\Theta} - 8\pi\mathcal{K}\mathcal{T}\,. \quad (14)$$

This highlights a physical competition: near GR ($\mathcal{K}\mathcal{T} \to 0$), expansion dominates ($\dot{\tau}_0 > 0$) requiring the causal Israel-Stewart formulation. In strong gravity regimes ($\mathcal{K}\mathcal{T} \to \infty$), the negative term dominates, making Eckart's theory a good approximation.

6. Exact Solutions: Consistency Checks

We validate the causal framework using known exact scalar-tensor solutions, computing explicit time evolution for the relaxation toward GR ($\mathcal{K}\mathcal{T} \to 0$).

Kasner vacuum

Using $a_i \propto t^{p_i}$ and $\phi \propto t^C$, the thermal parameter scales as:

$$\mathcal{K}\mathcal{T} = \frac{|C|}{8\pi t(1+C)}\,.$$

At late times ($t \to +\infty$), viscous components and $\mathcal{K}\mathcal{T}$ relax to zero, consistently recovering General Relativity as a **stable thermal equilibrium**.

O'Hanlon and Tupper (Power-law FLRW)

Using $a \propto t^q$ and $\phi \propto t^s$, we find:

$$\mathcal{K}\mathcal{T} = \frac{|s|}{8\pi t} \qquad \text{and} \qquad \mathcal{K} \propto t^{\frac{s-3}{2}}\,, \quad \mathcal{T} \propto t^{\frac{1-s}{2}}$$

This demonstrates the explicit analytical decoupling of $\mathcal{K}$ and $\mathcal{T}$. At any finite time t, $\mathcal{T}$ is identified as the **true primary parameter** driving the relaxation toward General Relativity.

Conclusions

- ✓ **Causality restored:** The IS map provides a physically viable thermal description of scalar-tensor gravity.
- ✓ **Cosmological impact:** The relaxation time τ_0 gives a practical tool to assess the validity of Eckart's approximation.
- ✓ **Outlook:** Extending the formalism to $f(\mathcal{R})$ gravity and further investigating its implications for cosmological singularities.

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