A new class of rotating charged black holes in the external Bertotti-Robinson (electro)magnetic field

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Introduction

Exact black hole spacetimes (such as Reissner–Nordström, Kerr, or Kerr–Newman) provide powerful models for investigations of the physical processes near the black holes. However, the incorporation of the external electromagnetic field creates complications. The first result in this direction is the Wald solution [1], where the asymptotically uniform magnetic field in a small field approximation around the Kerr solution was found (without the backreaction). The first models, when the backreaction was taken into account, are the Kerr–Melvin and Kerr-Newman–Melvin spacetimes [2]. These spacetimes describe black holes, immersed in the Bonnor-Melvin universe. However, these universes have some issues that do not allow one to use it for a realistic description of the magnetized black holes, namely (i) as one goes to infinity, the electromagnetic field decreases, while the gravitational field becomes stronger, (ii) both charged and uncharged particles cannot escape to infinity in the equatorial plane, (iii) these spacetimes are of algebraic type I (meaning that they radiate). To overcome these issues, we constructed a novel model of black holes, immersed in a uniform electromagnetic field [3,4].

Choosing an ansatz

To find a solution, we choose an ansatz

$$ds^2=\frac{1}{\Omega^2}\Big[-\frac{Q}{\rho^2}(\mathrm{d}t-p^2\mathrm{d}\varphi)^2+\frac{\rho^2}{Q}dr^2 + \frac{\rho^2}{P}dx^2+\frac{P}{\rho^2}(\mathrm{d}t+q^2\mathrm{d}\varphi)^2\Big] \tag{1}$$

with $\rho^2=q^2+p^2$, $Q(q)$, $P(p)$. An electromagnetic field is chosen in the form

$$\mathbf{A}=A_t\mathrm{d}t+A_\varphi\mathrm{d}\varphi. \tag{2}$$

This conformal-to-Carter spacetime is the most general type D spacetime with geodesic and shear-free principal null directions (PND) of the Weyl tensor that also satisfy some other geometrical constraints that can be found in [5].
The PNDs of the electromagnetic field in general are not aligned with the PNDs of the Weyl tensor (unlike the case of Plebanski-Demianski spacetime).

Solution

The only non-trivial solution with the non-aligned electromagnetic field is given by (proof can be found in [4,5])

$$\mathbf{A}=\frac{1}{4\bar{c}}\Big[\frac{\Omega_{,q}-\mathrm{i}\Omega_{,p}}{q+\mathrm{i}p}\mathrm{d}t + \Big(\frac{q^2\Omega_{,q}+\mathrm{i}p^2\Omega_{,p}}{q+\mathrm{i}p}-\Omega\Big)\mathrm{d}\varphi\Big]. \tag{3}$$

The metric functions are

$$Q=b_0+b_1q+b_2q^2+b_3q^3+b_4q^4, \tag{4}$$
$$P=a_0+a_1p+a_2p^2+a_3p^3+a_4p^4, \tag{5}$$
$$\Omega^2=(c_{00}+c_{01}q+c_{02}q^2)+(c_{10}+c_{11}q)p + c_{12}pq^2+(c_{20}+c_{21}q+c_{22}q^2)p^2. \tag{6}$$

The gauge freedom can be chosen to set

$$c_{00}=1,\quad c_{11}=-2, \tag{7}$$

and the field equations give a constraint

$$c_{02}=-c_{20}. \tag{8}$$

along with the explicit dependence of the parameters a_i and b_i on the coefficients c_{ij} (the explicit relations can be found in [4])

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Alternative parametrization

However, the original parametrization does not allow for the physical interpretation of the spacetime. By performing a set of transformations (see [4]), the metric may be written in the form

$$\mathrm{d}s^2=\frac{1}{\Omega^2}\Big[-\frac{\mathcal{Q}}{\varrho^2}\Big(\mathrm{d}t-\big[a\sin^2\theta+2(l+\omega\,x_0)(1-\cos\theta)\big]\mathrm{d}\varphi\Big)^2+\frac{\varrho^2}{\mathcal{Q}}\,\mathrm{d}r^2 + \frac{\varrho^2}{\tilde{\mathcal{P}}}\mathrm{d}\theta^2+\frac{\tilde{\mathcal{P}}}{\varrho^2}\sin^2\theta\Big(a\,\mathrm{d}t-\big[(r+r_0)^2+(a+l+\omega\,x_0)^2\big]\mathrm{d}\varphi\Big)^2\Big]. \tag{9}$$

Here $\mathcal{Q}$ is quartic in r, $\tilde{\mathcal{P}}$ is quadratic in $\cos\theta$, Ω^2 is quadratic in both r and $\cos\theta$ (for exact expressions see [4]), and $\rho^2=(r+r_0)^2+(a\,\cos\theta+l+\omega x_0)^2$. This solution depends on 7 parameters: the mass m, the Kerr a and Taub-NUT l parameters, acceleration α, complex parameter c (representing the strength of the external field), and parameter β representing duality rotation between two (not-independent) parameters r_0 and x_0: $\omega x_0=A+R\sin\beta$, $r_0=B+R\cos\beta$, where A, B, R are specific functions of m, a, l, α, $|c|$. The parameters r_0 and x_0 are related to the charge of the BH.

Kerr–Bertotti–Robinson spacetime

The most interesting subcase, out of various others, is the limit in which $\alpha\to 0$ while $2\alpha|c|=B=$ const. (with $r_0=0=x_0$). In this case, the metric becomes

$$\mathrm{d}s^2=\frac{1}{\Omega^2}\Big[-\frac{Q}{\rho^2}\big(\mathrm{d}t-a\sin^2\theta\,\mathrm{d}\varphi\big)^2+\frac{\rho^2}{Q}\,\mathrm{d}r^2+\frac{\rho^2}{P}\,\mathrm{d}\theta^2+\frac{P}{\rho^2}\sin^2\theta\,\big(a\,\mathrm{d}t-(r^2+a^2)\,\mathrm{d}\varphi\big)^2\Big], \tag{10}$$

where the metric functions are

$$\rho^2=r^2+a^2\cos^2\theta\,,\quad P=1+B^2\Big(m^2\frac{I_2}{I_1^2}-a^2\Big)\cos^2\theta\,,$$
$$Q=\big(1+B^2r^2\big)\Delta\,,\quad \Delta=\Big(1-B^2m^2\frac{I_2}{I_1^2}\Big)r^2-2m\frac{I_2}{I_1}r+a^2\,,$$
$$\Omega^2=\big(1+B^2r^2\big)-B^2\Delta\cos^2\theta\,,\quad I_i=1-\frac{i}{2}B^2a^2.$$

Its special cases are:

- $B=0$: Kerr spacetime
- $m=0$: Bertotti–Robinson spacetime
- $a=0$: Van den Bergh-Carminati solution [6]

The metric (10), thus can be called the Kerr–BR spacetime. Unlike the Ernst solution, the Kerr-BR black hole is of algebraic type D, and the external field is asymptotically uniform, as can be seen from the pictures in Fig. 1 at large distances from the center. Also, for any mass, there is an expulsion of the magnetic field-the so called black hole Meisner effect.

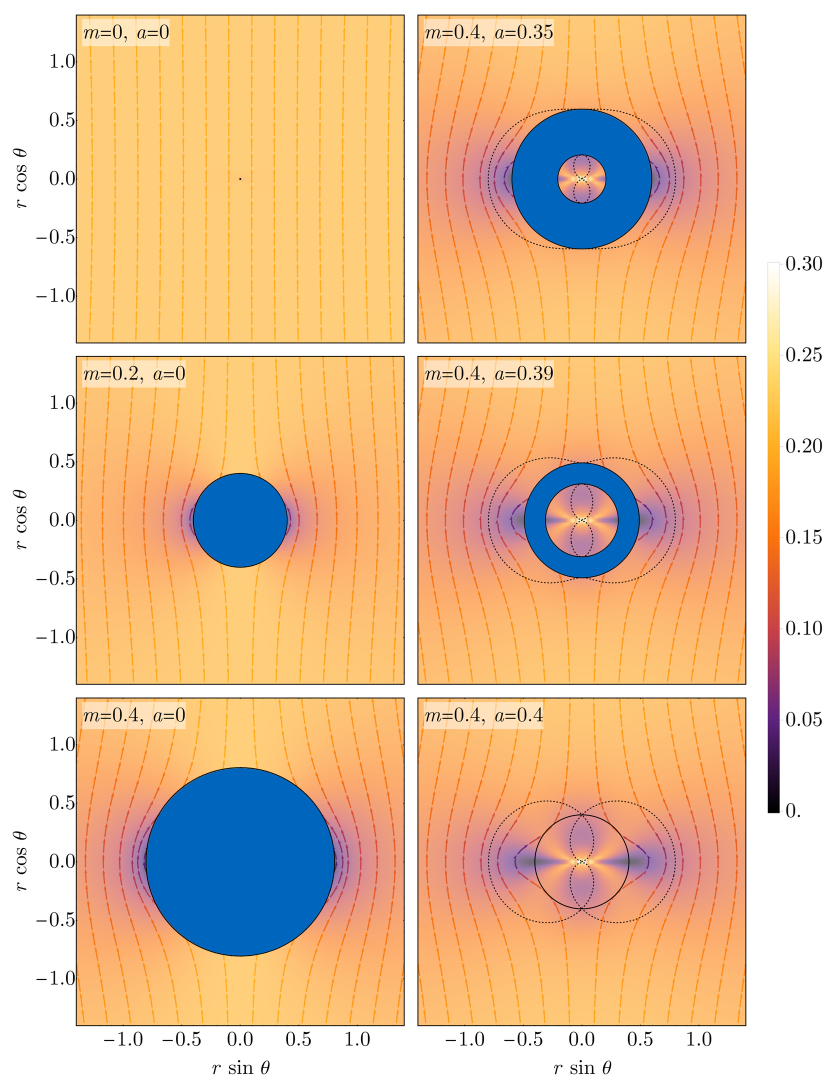

Figure 1: Magnetic field around the Kerr–BR black hole.

Referencies

[1]: R. M. Wald, Black hole in a uniform magnetic field, Phys. Rev. D **10**, 1680 (1974).
[2]: F. J. Ernst, Black holes in a magnetic universe, J. Math. Phys. **17**, 54 (1976).
[3]: J. Podolský, H. Ovcharenko, Kerr Black Hole in a Uniform Bertotti-Robinson Magnetic Field: An Exact Solution, Phys. Rev. Lett. **135** (2025) 181401.
[4]: H. Ovcharenko, J. Podolský, New class of rotating charged black holes with nonaligned electromagnetic field, Phys. Rev. D **112** (2025) 064076.
[5]: H. Ovcharenko, J. Podolský, Uniqueness of stationary axisymmetric type D black holes with non-aligned electromagnetic field, arXiv:2604.13202 (2026).
[6]: N. Van den Bergh and J. Carminati, Non-aligned Einstein–Maxwell Robinson–Trautman fields of Petrov type D, Class. Quantum Gravity **37** (2020) 215010.