# Geometrically Preferred Conformal to Kerr-NUT-AdS Spacetimes

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## Hidden symmetries and the Killing tower

The Kerr-NUT-AdS geometry in $D = 2n + \varepsilon$ dimensions reads

$$g = \sum_{\mu=1}^{n} \left[ \frac{U_\mu}{X_\mu} \mathrm{d}x_\mu^2 + \frac{X_\mu}{U_\mu} \Big( \sum_{j=0}^{n-1} A_\mu^{(j)} \mathrm{d}\psi_j \Big)^2 \right] + \varepsilon \, \frac{c_0}{A^{(n)}} \Big( \sum_{k=0}^{n} A^{(k)} \mathrm{d}\psi_k \Big)^2 ,$$

with $X_\mu = X_\mu(x_\mu)$ arbitrary: we work **off-shell** throughout.

It admits a **principal tensor**, a *closed* conformal Killing–Yano (CCKY) 2-form,

$$h^{\text{(KNA)}} = \mathrm{d}b \,, \qquad b = \tfrac{1}{2} \sum_{k=0}^{n-1} A^{(k+1)} \mathrm{d}\psi_k \,.$$

This single object generates the **whole tower of symmetries**. Its wedge powers are again CCKY forms; their Hodge duals are Killing–Yano forms:

$$h_p^{\text{(KNA)}} = \tfrac{1}{p!} \underbrace{h^{\text{(KNA)}} \wedge \cdots \wedge h^{\text{(KNA)}}}_{p \text{ times}} , \qquad k_{n-p}^{\text{(KNA)}} = * \, h_p^{\text{(KNA)}} \,.$$

Squaring gives the Killing tensors; the divergence of $h^{\text{(KNA)}}$ gives the primary Killing vector, and contraction gives the rest:

$$K_{ab}^{(j)} = \sum_{k=0}^{j} (-1)^k A^{(j-k)} \big(Q^k\big)_{ab} \,, \qquad Q_{ab} = h^{\text{(KNA)}}_{ac} h_b^{\text{(KNA)}c} ,$$

$$\xi^{(0)a} = \tfrac{1}{D-1} \nabla_b h^{\text{(KNA)}ba} = (\partial_{\psi_0})^a \,, \qquad \xi^{(j)a} = K^{(j)a}{}_b \, \xi^{(0)b} = (\partial_{\psi_j})^a \,.$$

## What survives a conformal rescaling?

Consider the Weyl-rescaled spacetime

$$\tilde{g}_{ab} = \frac{1}{\Omega^2} g_{ab} \,, \qquad \Omega = \Omega(x_1, \dots, x_n) \,.$$

Conformal Killing vectors and tensors carry over unchanged, and a conformal Killing–Yano form of rank $q$ (where $q = 2p$ for $h_p$) picks up a power of $\Omega$:

$$\tilde{\xi}^a = \xi^a \,, \qquad \tilde{K}^{ab} = K^{ab} \,, \qquad \tilde{\omega}_{a_1 \dots a_q} = \frac{1}{\Omega^{q+1}} \omega_{a_1 \dots a_q} \,.$$

The *conformal* tower is therefore always inherited.

**However, closure is not preserved.** The principal tensor degenerates into an ordinary CKY 2-form, $\mathrm{d}h_1 \neq 0$, and with it the construction above breaks down.

**Question:** for which $\Omega$ does the inherited *hidden* symmetry still generate *explicit* symmetries?

Answered in $D = 4$ in [1]. Here we go to $D = 6$.

## Killing vector candidates

The rescaled forms and the quadratic tensors built from them,

$$h_p = \frac{h_p^{\text{(KNA)}}}{\Omega^{2p+1}} \,, \qquad \big(Q_{h_p}\big)^a{}_b = \frac{1}{(2p-1)!} (h_p)^{ac_2 \dots c_{2p}} (h_p)_{bc_2 \dots c_{2p}} \,,$$

provide **two natural vectors**,

$$\xi^a = \tfrac{1}{D-1} \nabla_b h_1^{\,ba} \,, \qquad \eta^a = \tfrac{1}{D-1} \nabla_b k_1^{\,ba} \,,$$

and **eight more** obtained by contracting with the $Q$'s,

$$\xi_{h_p} = Q_{h_p} \cdot \xi \,, \quad \eta_{h_p} = Q_{h_p} \cdot \eta \,, \quad \xi_{k_p} = Q_{k_p} \cdot \xi \,, \quad \eta_{k_p} = Q_{k_p} \cdot \eta \,.$$

Demanding that each equals $a\partial_{\psi_0} + b\partial_{\psi_1} + c\partial_{\psi_2}$ yields a system of PDEs for $\Omega$.

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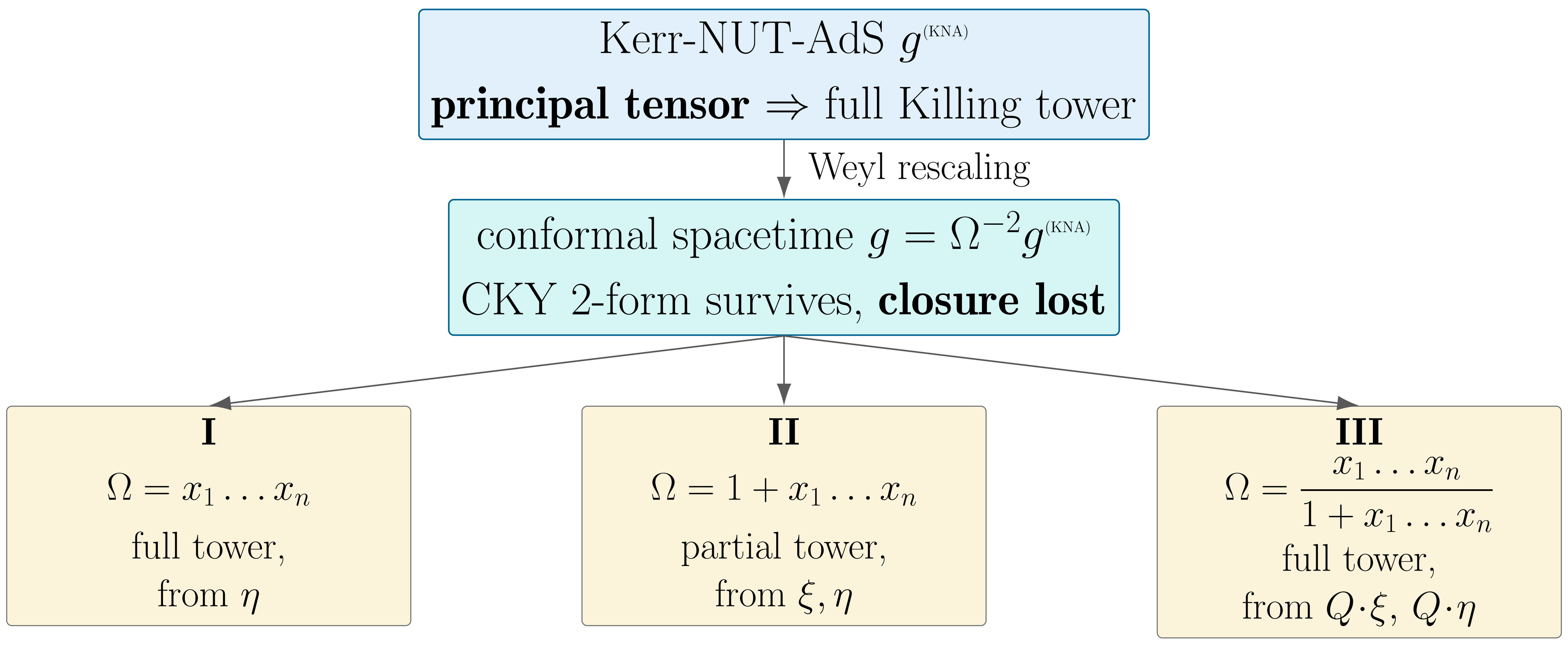
## References

[1] F. Gray, D. Kubizňák, H. Ovcharenko, and J. Podolský, "Hidden symmetries and separability structures of ovcharenko-podolský and conformal-to-carter spacetimes," *Phys. Rev. D*, vol. 113, p. 044050, 2026.




**Schema**: only three conformal factors retain a tower of explicit symmetries. Family **I** turns out to be Kerr-NUT-AdS itself, in a conformal frame.

## Three geometrically preferred conformal factors

Each solution comes with integration constants, but a rescaling $x_\mu = \lambda x'_\mu$ or an inversion $x_\mu \to 1/x'_\mu$ absorbs them. Every family thus reduces to a single representative:

| | Canonical form | Geometry | $n = 2$ limit |
|---|---|---|---|
| **I** | $\Omega = x_1 \dots x_n$ | Kerr-NUT-AdS in new coordinates | Carter |
| **II** | $\Omega = 1 + x_1 \dots x_n$ | analogue of Plebański–Demiański | $\Omega^{(PD)} = 1 - pq$ |
| **III** | $\Omega = \dfrac{x_1 \dots x_n}{1 + x_1 \dots x_n}$ | 'inverse' Plebański–Demiański | novel geometry [1] |

## Which candidate generates which isometry?

| Candidate | **I** $x_1x_2x_3$ | **II** $1 + x_1x_2x_3$ | **III** $\frac{x_1x_2x_3}{1+x_1x_2x_3}$ |
|---|---|---|---|
| $\xi = \frac{1}{5} \nabla \cdot h_1$ | $0$ | $\partial_{\psi_0}$ | — |
| $\eta = \frac{1}{5} \nabla \cdot k_1$ | $\partial_{\psi_2}$ | $\partial_{\psi_2}$ | — |
| $\eta_{h_1} = Q_{h_1} \cdot \eta$ | $\partial_{\psi_0}$ | — | $\partial_{\psi_0}$ |
| $\eta_{h_2} = Q_{h_2} \cdot \eta$ | $\partial_{\psi_1}$ | — | $\partial_{\psi_1}$ |
| $\xi_{k_1} = Q_{k_1} \cdot \xi$ | $0$ | — | $\partial_{\psi_2}$ |
| $\xi_{k_2} = Q_{k_2} \cdot \xi$ | $0$ | — | $\partial_{\psi_1}$ |
| $\xi_{h_1}, \ \xi_{h_2}$ | $0$ | — | — |
| $\eta_{k_1}, \ \eta_{k_2}$ | — | — | — |

- **I** $\xi$ vanishes, $h_1$ is a genuine Killing–Yano tensor, and the *full tower* comes from the dual side. An inversion $x_\mu \to 1/x_\mu$ with $\psi_0 \leftrightarrow \psi_2$ shows this is Kerr-NUT-AdS in different coordinates.
- **II** generated by $\xi, \eta$ directly, as in $D = 4$, but only *two of three* isometries appear. $\xi_{k_2} = \Omega^{-2} \partial_{\psi_1}$ points along the missing one without being Killing.
- **III** neither $\xi$ nor $\eta$ is Killing, yet the *full tower* reappears from the contractions with $Q$.

## Remarks and outlook

- **In few words:** among all spacetimes conformal to Kerr-NUT-AdS, exactly three families retain a tower of explicit symmetries generated by the hidden one.
- **Off-shell:** with arbitrary $X_\mu(x_\mu)$ these are statements about geometric structure, not about solutions of field equations. In $D = 4$ family **II** is Einstein–Maxwell, while **III** cannot be vacuum [1].
- **Next:** separability in the two new frames, and whether further families appear for $D \geq 8$.