Two rings to rule them all? Observing double Einstein rings in an exact solution of $f(R)$ gravity

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Abstract

We present the **first implementation** of a **modified theory of gravity** into the s**ource reconstruction and lens modelling formalism** of **PyAutoLens** [1]. We implement a **deflection angle** that was obtained from the **derivation of a general framework** for the study of strong lensing in **$f(R)$ gravity** using the **1 + 1 + 2 covariant approach** [2]. This **deflection angle** emerges from an **exact spherically symmetric solution of R^n gravity**, and **uniquely predicts the formation of a double Einstein ring** from a **single source galaxy** [3]. In this work, we **illustrate the statistically competitive nature of $f(R)$ gravity** in the analysis of galaxy-galaxy strong lensing. Furthermore, we discuss the **dependence** of the **observed magnification** of the lensed images **to the particular theory of gravity,** as well as **motivate** for the **broader introduction** of **physically-motivated alternative theories of gravity** into the **lens modelling and source reconstruction analysis** of strongly lensed systems.

A whirlwind of theoretical background

Strong gravitational lensing offers an **intimate probe** into the **underlying physical properties** of the constituent lens and source galaxies. This is often achieved through the means of **ray tracing** and **lens modelling** techniques [4, 5, 6, 7, 8, 9, 10].

We implemented the **deflection angle** from the exact solution (ECS hereafter) into **PyAutoLens**. This is the **first known implementation** of a **modification to GR-based deflection angles** into the **lens modelling framework**. The deflection angle is given as:

$$\hat{\alpha} = \frac{4GM}{c^2}\left[\frac{1}{r_0} - \frac{1}{2r_1} - \frac{1}{2r_2} + (n-1)\left(\frac{1+\ln r_0}{r_0} - \frac{1+\ln r_1}{2r_1} - \frac{1+\ln r_2}{2r_2}\right)\right]\hat{\xi}\,,$$

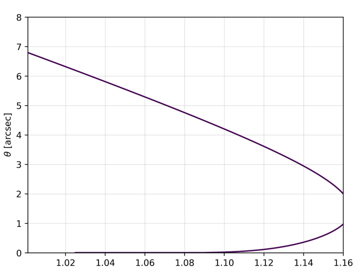

Figure 1: Einstein radius [arcsec] as a function of n.

This deflection angle **uniquely predicts** the **formation** of **two Einstein rings** for **some values of *n***, see Figure 1.

In order to implement a different theory of gravity into PyAutoLens, you need the **deflection angle** and a way to **numerically describe** the **distances** between the source and lens galaxies.

The PyAutoLens perspective

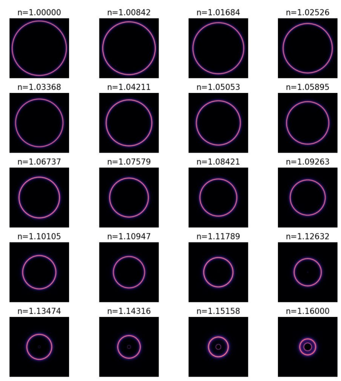

Figure 2: Simulated behaviour of Einstein rings for increasing values of n. Note the emergence of a faint second ring for $n > 1.14$.

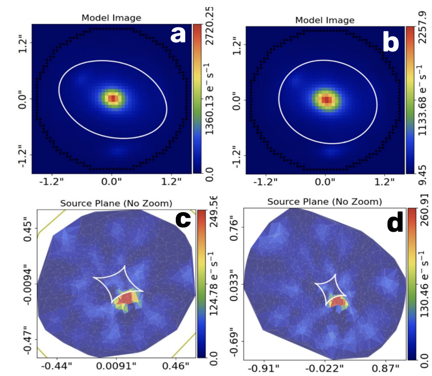

Figure 3: The GR power-law (a) and ECS (b) model reconstruction of an HST-selected strongly lensed system [11, 12, 13]. The source reconstruction for the same system is shown for the GR power-law (c) and ECS (d) cases.

Table 1: Model comparison between best-fit GR-power law and ECS lens models and source reconstruction

Best-fit Model	$\theta_{E,\mathrm{observed}}$ [″]	AIC	BIC
GR Power Law	$0.987^{+0.032}_{-0.027}$	15083.84	15127.42
Exact Clifton Solution	$1.086^{+0.008}_{-0.013}$	15197.76	15235.89

Statistically competitive with GR??

Magnification as a function of theory?

Figure 4: Simulated double Einstein ring for $n = 1.15$.

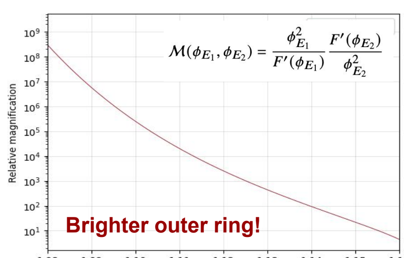

Figure 5: Relative magnification of the outer ring vs the inner ring as a function of n.

Aside from the appearance of two rings, this **theory also predicts** that the **two rings** will be **magnified differently**. This is a quirk that **would have been missed** entirely if this theory **were not modelled,** further **motivating** for the **inclusion** of **modified theories of gravity** in the **broader framework**.

The general form of the magnification is:

$$\mu = \left[1 - \frac{1}{\phi^4} + \frac{\gamma}{\phi^4}\right],$$

in this case, **γ is a function of *n*** and the various parameterizing **constants specific to the system** of the lens and source galaxies.

Future work?

- **Generalising this approach** to something that is more **theory agnostic**
- Investigating **whether PyAutoLens can differentiate between the various metrics** of a given theory?
- Express deflection angles of modified theories of gravity in a **more physically grounded** manner

References

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