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Regular Black Holes in Asymptotic Safety

Solutions, dynamical collapse, perturbations and phenomenology

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1 Asymptotic Safety & Static Regular Black Holes

Asymptotic Safety (AS) conjectures a UV-attractive non-Gaussian fixed point (NGFP) for gravity, making the quantum theory predictive at arbitrarily high energies. Near the NGFP the Newton coupling runs with the RG scale, G(k), weakening gravity at short distances. Promoting k to a function of position (RG improvement) turns this running into a quantum-corrected, singularity-free metric f(r) = 1 − 2G(r)M/r.

$$G(k) = \frac{G_0}{1+\omega G_0 k^2}$$

General criteria for the cutoff k(r)

- Coordinate-independent: built from diffeomorphism-invariant quantities (proper distances, curvature scalars).
- Compatible with the symmetries of the underlying spherically symmetric spacetime.
- Different, equally admissible choices of k(r) generate different improved solutions.

Bonanno–Reuter (2000)

Cutoff tied to the proper radial distance, k(r) ~ ξ/d(r). Regular de Sitter core as r→0, Schwarzschild recovered at large r. Horizonless below a critical mass.

$$f_{BR}(r) = 1 - \frac{2MG_0 r^2}{r^3 + \tilde{\omega} G_0\left(r + \frac{9}{2}G_0 M\right)}$$

Hayward

Cutoff tied to the Kretschmann scalar K, $k^2 = \alpha\sqrt{K}$. Reproduces the well-known Hayward metric; two horizons for γ below its critical value.

$$f_{Hay}(r) = 1 - \frac{2r^2/M^2}{r^3/M^3 + \gamma}$$

Dymnikova

Recursive improvement removes the background-dependence of the cutoff identification. de Sitter core; the horizon disappears above a critical length scale l.

$$f_{Dym}(r) = 1 - \frac{2M}{r}\left(1 - e^{-\frac{r^3}{2l^2 M}}\right)$$

Critical parameters (Planck / mass units)

Model	Critical parameter	Value
Bonanno–Reuter	M_{cr}	3.503
Hayward	γ_{cr}	32/27 ≈ 1.185
Dymnikova	l_{cr}	≈ 1.138 M

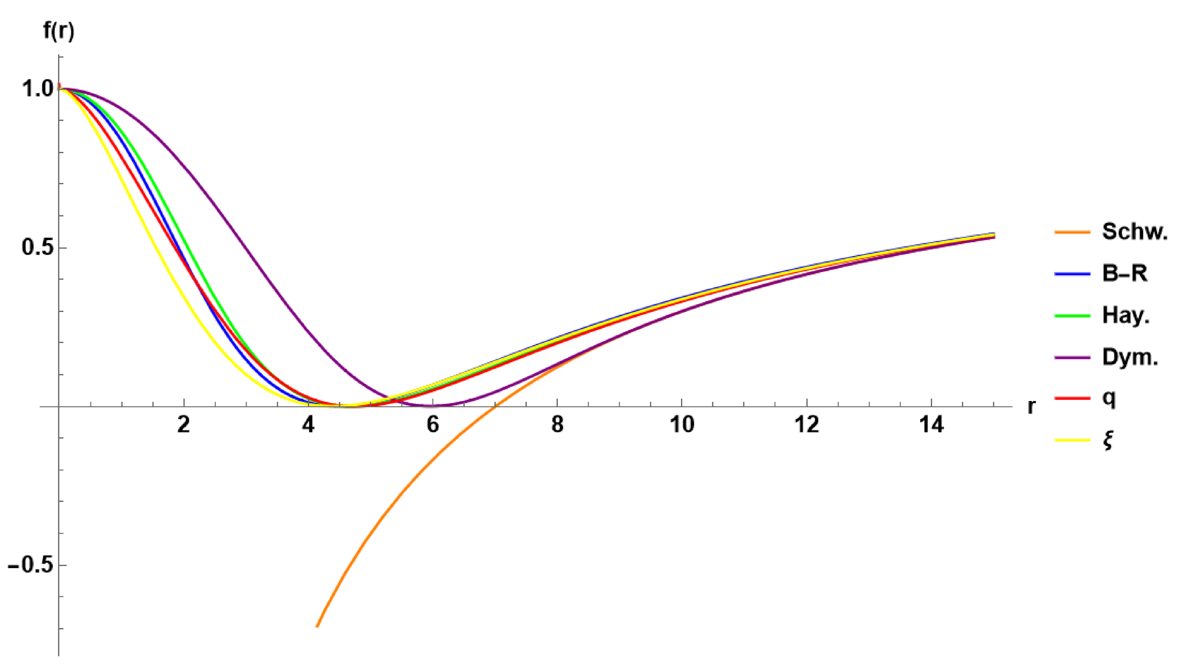

Fig. 1 — Metric functions f(r) for the three regular BH solutions at their critical parameter (M = 3.503). All curves converge to Schwarzschild within a few horizon radii; quantum corrections are confined to the near-horizon region.

Common features of all static solutions

- Schwarzschild geometry is exactly recovered as r → ∞.
- A regular core (de Sitter-like) replaces the classical curvature singularity at r = 0.
- A single critical parameter controls the horizon count: two horizons, one extremal horizon, or none.

2 Dynamical Collapse from Proper-Time Flow

Static RG-improved metrics lack a formation process. We consider a quantum-gravity-improved Oppenheimer–Snyder–Datt dust collapse: the matter–gravity coupling χ(ε) in the action makes Newton's constant run with the fluid's proper energy density ε.

$$S = \frac{1}{16\pi G_0}\int d^4x\sqrt{-g}\,[R + 2\chi(\varepsilon)\mathcal{L}]$$

The proper-time (PT) flow for the Einstein–Hilbert truncation has a UV fixed point. The solutions obtained by integrating the beta functions is independent of gauge and parametrization choices; only the regularization scheme (B or C) fixes the functional form of G(ε).

$$G^{(B)}(\varepsilon) = \frac{1}{\frac{\varepsilon}{2q} + \sqrt{1+\left(\frac{\varepsilon}{2q}\right)^2}} \qquad G^{(C)}(\varepsilon) = \frac{1}{1+\varepsilon\xi}$$

Scheme B (ε = 1) *Scheme C (ε = 0)*

Matching the interior FLRW dust cloud to the static exterior via the Israel junction conditions on the boundary Σ fixes the exterior metric f(R), which depends on the regularization scheme chosen for G(ε):

Scheme B (ε = 1) — this work:

$$f(R) = \frac{3M_0^2 + qR^4 - M_0\sqrt{9M_0^2 + q^2R^6}}{qR^4} + \frac{2}{3}qR^2\,\mathrm{arctanh}\left(\frac{\left(q - \sqrt{q^2 + \frac{9M_0^2}{R^6}}\right)R^3}{3M_0}\right)$$

Scheme C (ε = 0):

$$f(r) = 1 - \frac{r^2}{3\xi}\log\left(1 + \frac{6M\xi}{r^3}\right)$$

In Scheme B the collapse never reaches a = 0: the scale factor freezes at late times, $a(t) \sim e^{-\frac{qt^2}{4}}$, and the classical spacelike singularity is avoided.

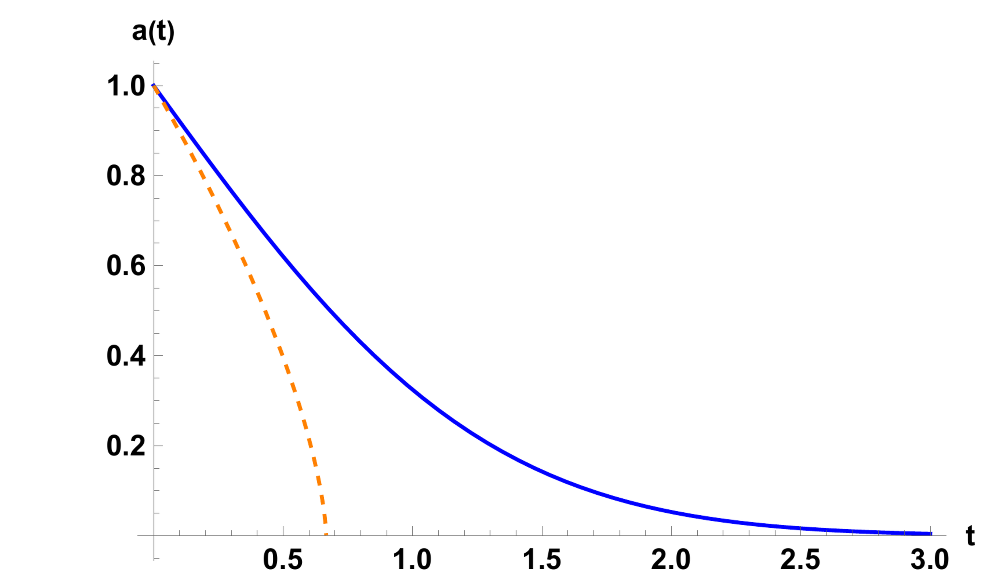

Fig. 2 — Scale factor a(t): regular collapse (q = 1.40, blue) vs. classical OSD collapse (dashed). The singularity a→0 is never reached.

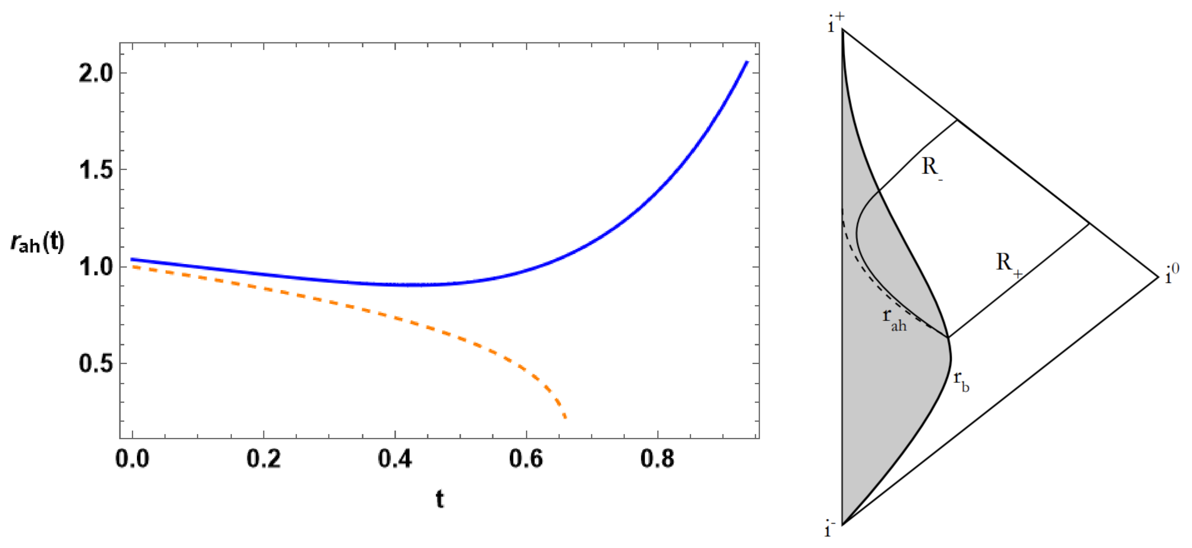

Fig. 3–4 — Apparent horizon $r_{ah}(t)$ (left): quantum effects halt and reverse its collapse. Penrose diagram (right): trapped region forms, no spacelike singularity.

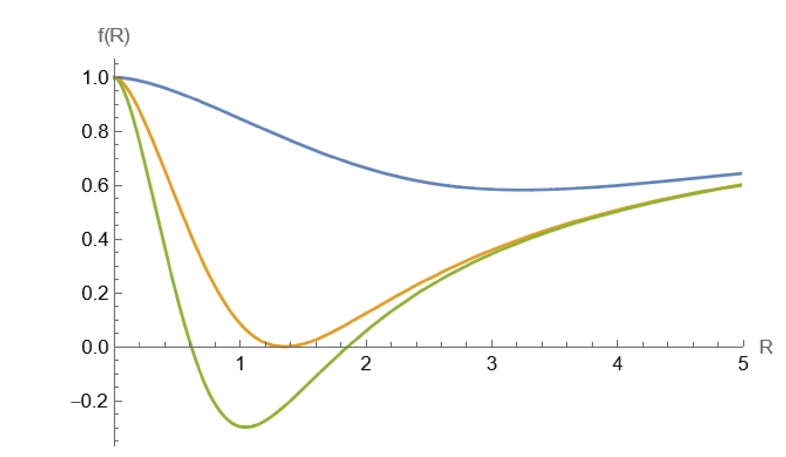

Fig. 5 — Exterior metric f(R), Scheme B, M = 1: two horizons for $q > q_{cr} \approx$ 1,37, one at criticality, none below.

Critical parameters (mass units, $G_0 = 1$)

Scheme	Parameter	Critical value
B (ε = 1)	q_{cr}	≈ 1.37
C (ε = 0)	ξ_{cr}	≈ 0.46

Why it matters

- Dynamical (collapse) realization of a regular black hole within Asymptotic Safety, complementing the static RG-improved metrics.
- The running Newton constant is independent of gauge and field-parametrization choices — only the regularization scheme matters.
- A trapped region and apparent/event horizons still form, but the classical spacelike singularity is replaced by a regular, everlasting core.

3 Perturbations, Hawking Radiation & Shadows

Axial gravitational perturbations are modelled as sourced by an effective anisotropic fluid, reducing to a Schrödinger-like master equation for the Regge–Wheeler variable Ψ, with an effective potential V(r) built from f(r) and its derivative:

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V(r)]\Psi = 0$$

QNM frequencies from three independent methods (WKB+Padé, Leaver continued fraction, time-domain + Prony) agree to < 0.4%. Deviations from Schwarzschild grow with overtone number n — the "outburst of overtones" — and peak near the critical parameter; the late-time ringdown decays into the universal Price power-law tail Ψ ~ t^{-7}, confirming stability.

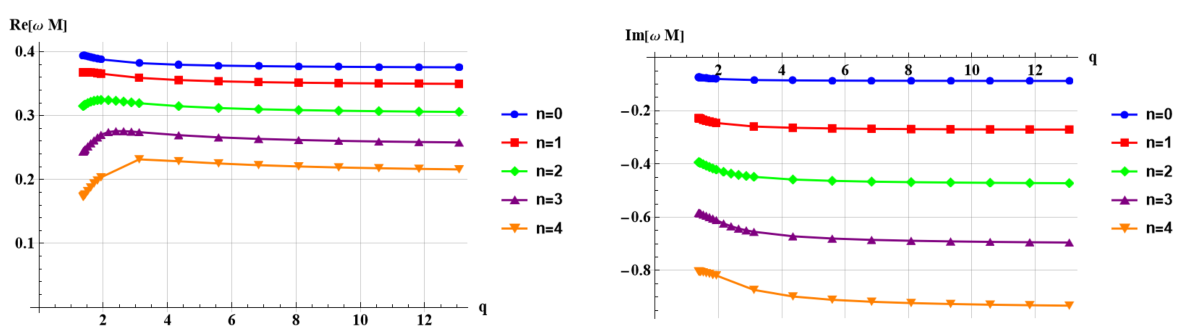

Fig. 6 -7 — Real (left) and imaginary (right) l = 2 overtone frequencies (n = 0–4, Leaver) vs. q: higher overtones deviate far more from Schwarzschild.

Scalar QNM frequencies near criticality (l=1, mass units)

Model	Re ω (n=0)	Im ω (n=0)	Re ω (n=3)	Im ω (n=3)
Schwarzschild	0.29293	−0.09766	0.20326	−0.78830
Hayward	0.30562	−0.08559	0.15811	−0.70061
Dymnikova	0.28874	−0.09468	0.15900	−0.96400
Scheme B	0.30598	−0.08300	0.13019	−0.70745
Scheme C	0.31052	−0.07986	0.14249	−0.67509

The fundamental mode (n=0) stays close to Schwarzschild for every model, but by n=3 the values visibly increase— deviations up to 36% (Re) and 22% (Im), the "outburst of overtones."

Hawking radiation

The semiclassical Hawking temperature $T_H = f'(r_H)/4\pi$ is systematically suppressed as regular solutions approach extremality — pointing to cold, long-lived Planck-mass remnants relevant for primordial black holes as dark matter component.

$$\frac{d^2N}{dt\,d\omega} = \frac{1}{2\pi}\sum_{lm}\frac{n_i\Gamma^s_{lm}(\omega)}{e^{\omega/T_H} \pm 1}$$

Model	T_H (mass units)
Schwarzschild	0.0398
Bonanno–Reuter	0.00071
Hayward	0.00369
Dymnikova	0.00291
Scheme B	0.00472
Scheme C	0.00345

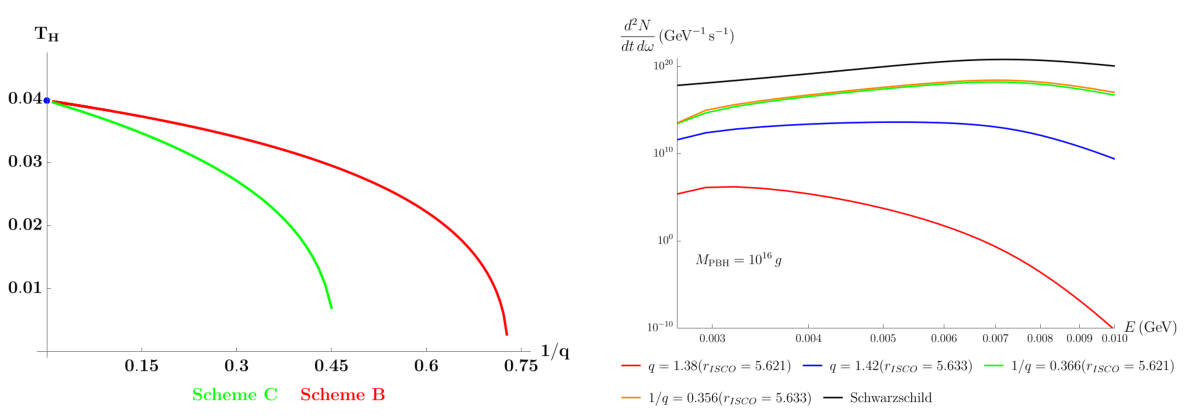

Fig. 8 — Left: T_H vs 1/q for both PT schemes, vanishing continuously at extremality. Right: photon emission spectrum for a $M_{PBH} = 10^{16}$g primordial black hole — near-critical q strongly suppresses and reshapes the flux relative to Schwarzschild.

Shadow radius

The shadow radius R_{Sh}, obtained from the photon-sphere condition, always shrinks relative to Schwarzschild — yet remains compatible with the EHT-inferred window (4.8–5.2 M) for every model considered.

Model	R_{Sh}
Schwarzschild	5.196
Bonanno–Reuter	4.833
Hayward	4.917
Dymnikova	5.196
Scheme B	4.940
Scheme C	4.866

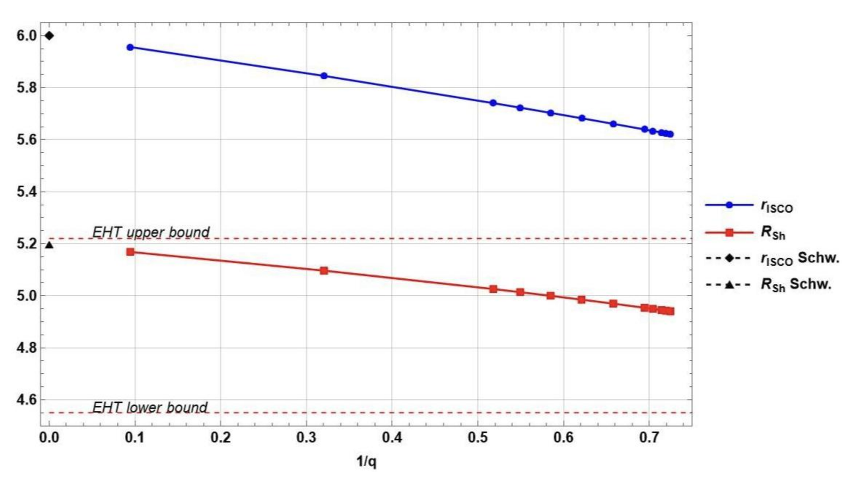

Fig. 9 — ISCO and shadow radius vs 1/q (Scheme B) compared to EHT bounds (dashed): all values lie within the observational window.

References: - A. Spina, Int. J. Gravit. Theor. Phys. 1(1), 8 (2025), arXiv:2510.14552 ·
- A. Bonanno, R. A. Konoplya, G. Oglialoro, A. Spina, JCAP12(2025)042 ·

Conclusions & Outlook

- All AS-inspired regular black holes recover Schwarzschild at large r, with quantum corrections confined to a few horizon radii.
- The dust-collapse model built from the proper-time flow avoids the classical singularity dynamically: the scale factor never vanishes and the apparent horizon halts and re-expands.
- QNM overtones are far more sensitive to the near-horizon quantum structure than the fundamental mode — a distinctive "sound of the event horizon" signature.
- Hawking evaporation is strongly suppressed near extremality while shadow sizes stay within current EHT bounds, leaving these models observationally viable and testable with next-generation facilities (ET, LISA, EHT).