Thermodynamics of AdS black holes

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BH Thermodynamics began w/ the Schwarzschild and Kerr black holes. These exhibit the peculiarity that the specific heat is negative (and other problems due to the ergosphere). Their thermodynamics (canonical ensemble) is not properly defined.

The situation improves if we put the system in a box. AdS provides a covariant box, and also allows for an interpretation in terms of a dual quantum field theory in one less dimension, through the AdS/CFT correspondence.

Black holes in AdS exhibit much richer and cleaner thermodynamic structure.

Also the calculation of thermodynamic quantities can be done w/ elegant methods motivated by holographic renormalization: counterterm subtraction.

Consider AdS_{n+1} gravity $R_{ij} = -\frac{n}{L^2} g_{ij}$

Solved by

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n-1}$$

$$f(r) = 1 - \frac{\mu}{r^{n-2}} + \frac{r^2}{L^2}$$

[could have $k - \frac{\mu}{r^{n-2}} + \frac{r^2}{L^2}$
$k = \pm 1, 0$
but won't do it]

If $\mu = 0$ this is AdS in global coordinates

Boundary at $r \to \infty$ is $\mathbb{R}_t \times S^{n-1}$

For $n=2$ we get the BTZ black hole (if $\mu > 1$)

This case is different than $n > 2$, which are all qualitatively the same. Henceforth we'll only do $n = 4$, ie AdS_5 black holes.

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2}$$

Horizon at $f(r_+) = 0$

$$\mu = \frac{r_+^2 (r_+^2 + L^2)}{L^2}$$

(convenient to use r_+ as parameter instead of μ)

AdS "compresses" the size of black holes:

$$r_+^2 = \mu - \frac{r_+^4}{L^2} < \mu$$

AdS has a length scale L, so we'll get different classes of bh's depending on whether

$r_+ \gtrless L$ small and large AdS bh's

(in $D=3$ only large AdS bh's)

$+ < -$

(in $D=3$ only large AdS bh's)

Small: $r_+ \ll L \Rightarrow r_+^2 \simeq \mu$ like in AF space

Large: $r_+ \gg L \Rightarrow r_+^4 \simeq \mu L^2 \qquad f(r) \simeq \frac{r^2}{L^2} - \frac{\mu}{r^2}$

$\approx$ planar black brane in AdS

Temperature (from surface gravity, or from Euclidean regularity)

$$T = \frac{1}{\beta} = \frac{f'(r_+)}{4\pi} = \frac{2r_+^2 + L^2}{2\pi L^2 r_+}$$

We now want to compute the free energy.

There are two ways:

- Quick, pedestrian:

  Mass M from asymptotic formula

  $$M = \frac{3\pi}{8G}\mu = \frac{3\pi}{8G}\frac{r_+^2(r_+^2+L^2)}{L^2}$$

  (can be derived from linearized analysis around AdS)

  Entropy from Area:

  $$S = \frac{A_+}{4G} = \frac{\pi^2}{2G} r_+^3$$

  Then $F = M - TS = \frac{\pi}{8GL^2}\left(-r_+^4 + r_+^2 L^2\right)$

- Sophisticated, "ab-initio" derivation from Euclidean path integral, w/ counterterm subtraction

In this case we compute the Euclidean action for the solution w/ $t \to i\tau$ $\quad \tau \sim \tau + \beta$

$$I = \underbrace{I_{EH}}_{\text{bulk}} + \underbrace{I_{YGH} + I_{\text{counterterm}}}_{\text{boundary}}$$

$$I_{EH} = -\frac{1}{16\pi G} \int_{\mathcal{M}} d^5x \sqrt{g} \left(R + \frac{12}{L^2} \right)$$

$$I_{YGH} = -\frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^4x \sqrt{h}\, K$$: well-defined variational problem w/ fixed bdry metric
NOT a counterterm

$$I_{\text{counterterm}} = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{h} \left(\frac{3}{L} + \frac{L}{4} \mathcal{R} \right)$$

↳ boundary scalar curvature

↳ depends only on intrinsic bdry geometry

Removes divergences that arise from having an infinite space w/ ∞ volume

E.g. bulk term, use $R = -\frac{20}{L^2}$ $\qquad R + \frac{12}{L^2} = -\frac{8}{L^2}$

$$I_{EH} = +\frac{1}{16\pi G} \int_{\mathcal{M}} d^5x \sqrt{g}\, \frac{8}{L^2} = \frac{\mathrm{Vol}(\mathcal{M})}{2\pi G L^2}$$

Introduce a long-distance regulator $r = \hat{R} \gg r_+, L$ expand for large $\hat{R}$. Divergences cancel out and finally take $\hat{R} \to \infty$

$\Omega_3 = 2\pi^2$

One finds

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$$I = \frac{\beta\pi}{8GL^2}\left(-r_+^4 + r_+^2 L^2 + \frac{3}{4}L^4\right) = \beta F$$

$$F = \underbrace{\frac{\pi}{8GL^2}\left(-r_+^4 + r_+^2 L^2\right)}_{\text{like above}} + E_0$$

$E_0 = \frac{3\pi}{32G}L^2$: independent of r_+ (μ)
Casimir energy of CFT in S^3
Doesn't affect thermo

Apply conventional thermo:

$$E = \partial_\beta I = \frac{3\pi}{8G}\mu + E_0 = M + E_0$$

$$S = (\beta\partial_\beta - 1)I = \frac{\pi}{2G}r_+^3 = \frac{A_H}{4G}$$

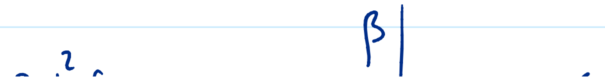
Note: at high T, w/ $r_+ \gg L$ we get

$$T \simeq \frac{r_+}{\pi L^2}$$

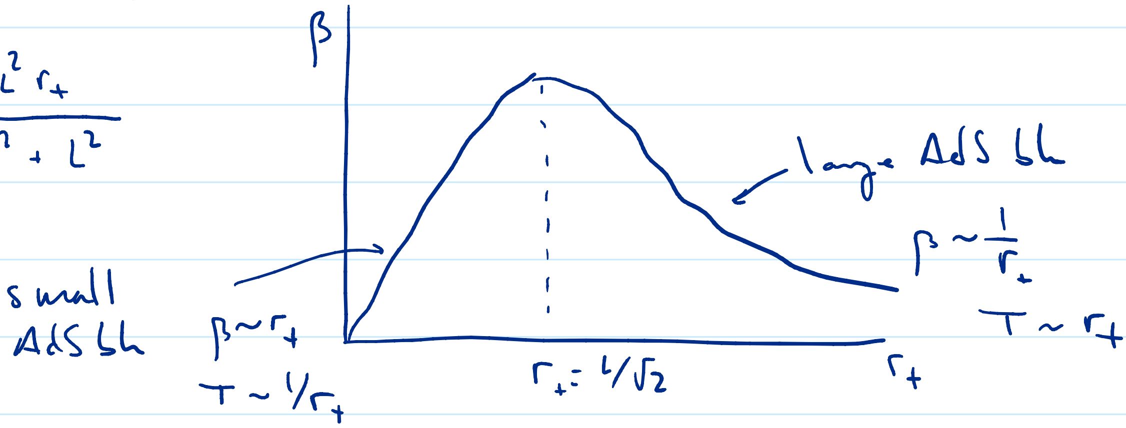
$$\left.\begin{array}{l} F \sim T^4 \\ E \sim T^4 \\ S \sim T^3 \end{array}\right\}$$

like a conformal (massless) theory at finite temperature in <u>4 dimensions</u>

Phase diagram



$$\beta = \frac{2\pi L^2 r_+}{2 r_+^2 + L^2}$$

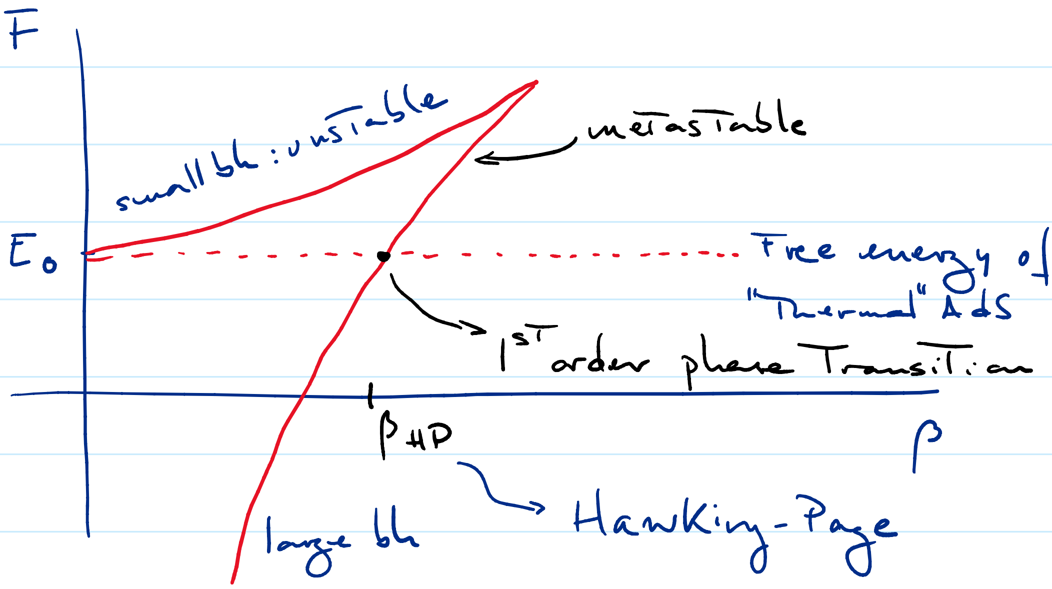


Specific heat: $C = T\frac{\partial S}{\partial T} = -\beta \frac{\partial S}{\partial \beta} = -\frac{3\pi^2}{2G} r_+^2 \frac{\partial r_+}{\partial \beta}$

$\frac{\partial r_+}{\partial \beta} > 0 \Rightarrow C < 0$: small bhs thermo unstable

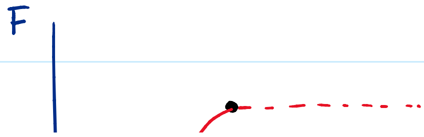
$\frac{\partial r_+}{\partial \beta} < 0 \Rightarrow C > 0$: large bhs thermo stable

In canonical ensemble, F is minimized:

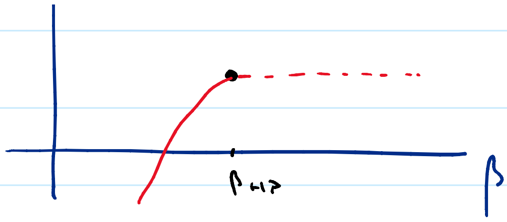


Phase Transition for $r_+ = L$ $\quad \beta_{HP} = \frac{2\pi}{3} L$

Phases:



Phases:



low T: $0 \leq T < T_{HP}$: Thermal AdS_5
(w/ Thermal radiation gas, whose energy and entropy are negligible)

$T > T_{HP}$: large black hole
Represents deconfined phase of CFT at finite Temperature

At large T, the entropy of the black hole overwhelms the suppression $e^{-\beta E}$.

Latent heat at transition is $T_{HP} S_{HP}$

- More complex phase structures are possible:
  - add charge: RN-AdS : chemical potential in the CFT
    finite "baryon" density
  - vary Λ : "extended thermodynamics"
    "black hole chemistry"
    Enthalpy $H = E + PV$
    (P from Λ)

AdS black brane Thermodynamics

Let us now study the properties of very large AdS black holes.

Starting from the solution above, we scale

$r \to \lambda r$ $\qquad\qquad \mu \to \lambda^4 \mu$

$t \to t/\lambda$

$L^2 d\Omega_3 \to \frac{1}{\lambda^2} d\vec{x}_{(3)}^2$ (stereographic coords around a point)

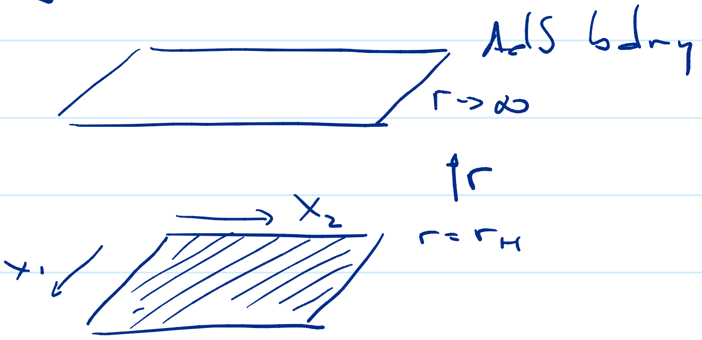
and send $\lambda \to \infty$: very large black hole.

We obtain a solution with a planar horizon:

$$ds^2 = -V(r)\,dt^2 + \frac{dr^2}{V(r)} + \frac{r^2}{L^2} d\vec{x}_{(3)}^2$$

$$V(r) = \frac{r^2}{L^2} - \frac{\mu}{r^2}$$

This is the AdS_5 black brane.



The horizon is at $r_+ = (\mu L^2)^{1/4}$

and the temperature $T = \frac{r_+}{\pi L^2}$

The entropy and energy now become infinite because the planar horizon has infinite extent.

because the planar horizon has infinite extent.

But we can define an energy density and entropy density:

$$s = \frac{1}{4G}\frac{r_+^3}{L^3} = \frac{\pi^3 L^3}{4G} T^3$$

(like a "photon gas" in 3+1 dimensions

$$\rho = \frac{3}{16\pi G}\frac{r_+^4}{L^5} = \frac{3}{16\pi G}\frac{\mu}{L^3} = \frac{3\pi^3 L^3}{16 G} T^4$$

Note $\quad d\rho = T\,ds$

Since this black brane is an extended object, we expect not only an energy density, but also pressure components along the spatial directions x^i.

Energy density is read from g_{tt}

Pressure " " " g_{ii}

Momentum " " " g_{ti}

From thermodynamics we expect

$$E + PV = TS \quad \text{ie} \quad \rho + P = Ts$$

so we can obtain $P = Ts - \rho$

$$P = \frac{\rho}{3} = \frac{\pi^3 L^3}{16 G} T^4 \; : \text{ equation of state}$$

of radiation fluid in 3+1

(conformally invariant)

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These are the thermodynamic properties of the quantum state that is dual to the AdS black brane: a thermal radiation, scale-invariant plasma.

- AdS_5/CFT_4 dictionary for CFT_4: $\mathcal{N}=4$ $SU(N)$ SYM

$$N^2 = \frac{\pi L^3}{2G}$$

When the quantum theory is free, one has, from standard thermodynamic analysis, that the thermal equilibrium state has

$$p = \sigma T^4 \quad \text{w/} \quad \sigma = \frac{\pi^3}{30}\Big(\underset{\substack{\uparrow \\ \text{scalars}}}{n_0} + \frac{7}{4}\underset{\substack{\uparrow \\ \text{fermions}}}{n_{1/2}} + 2\underset{\substack{\uparrow \\ \text{vectors}}}{n_1}\Big)$$

for $\mathcal{N}=4$ $SU(N)$ SYM $\quad n_0 = 6N^2$

$$n_{1/2} = 4N^2$$

$$n_1 = N^2$$

so $p_{free} = \frac{\pi^3}{30} N^2 \, 15 \, T^4 = \frac{\pi^3 N^2}{2} T^4$

The gravity calculation gives

$$p_{grav} = \frac{3\pi^3}{16G} L^3 T^4 = \frac{3\pi^3 N^2}{8} T^4$$

So $\rho_{free} = \frac{4}{3} \rho_{grav}$: almost the same.

The $\frac{4}{3}$ difference is because the gravitational calculation is dual to a strongly coupled regime of the quantum theory.