Black holes in D >=5

viernes, 16 de septiembre de 2022 20:23

In $D \geq 5$ we'll have black holes that are more similar to the 4D ones: asymptotically flat, but also with richer behavior.

While in 4D the classification of black holes for $R_{\mu\nu}=0$ (and others) is complete, due to the uniqueness theorems, in higher D the classification is incomplete, especially in $D \geq 6$. Indeed, classifying all black holes may not be a good nor interesting idea. Instead, it's better to try to understand what rules their behavior and main properties, and for this we do have quite good understanding.

We'll begin extending the 4D solutions; later we'll discuss black holes that don't have 4D counterparts.

Newtonian potential in D dimensions

$$\Phi \sim \frac{1}{r^{D-3}}$$

Schwarzschild-Tangherlini ('63):

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}$$

$$f(r) = 1 - \frac{\mu}{r^{D-3}}$$

Mass $M \propto \mu$. Horizon at $r_H = \mu^{1/D-3}$ $\quad M = \frac{(D-2)\Omega_{D-2}}{16\pi G}\mu$

It extends easily to (A) dS and Reissner-Nordström:

(A) dS

$$f(r) = k - \frac{\mu}{r^{D-3}} + \frac{r^2}{L^2} \qquad \Lambda \propto -\frac{1}{L^2}$$

$k=1$ dS $\quad$ $k = \pm 1, 0$ AdS

Mink

RN:

$$f(r) = 1 - \frac{\mu}{r^{D-3}} + \frac{q^2}{r^{2(D-3)}} \qquad \text{charge } Q \propto q$$

Does this simple trick apply also for rotating bhs?

Notice that in $D \geq 5$ there can be rotation in several independent planes:

$$\underbrace{x_1, x_2}_{(r_1, \phi_1)}, \underbrace{x_3, x_4}_{(r_2, \phi_2)}, \underbrace{x_5, x_6}_{(r_3, \phi_3)}, \dots$$

Flat space $ds^2 = \underbrace{dr_1^2 + r_1^2 d\phi_1^2}_{J_1} + \underbrace{dr_2^2 + r_2^2 d\phi_2^2}_{J_2} + \dots$

$\left\lfloor \frac{D-1}{2} \right\rfloor$ independent rotation planes (maximal Cartan Torus of $SO(D-1)$)

Let's begin with rotation in a single plane.
These solutions are simpler, and also exhibit more novel physics

Miraculously, The Trick does work!

Kerr $\frac{\mu}{r} \to \frac{\mu}{r^{D-3}}$ and add angular directions

Myers-Perry black holes ('86) (w/ single spin)

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + (r^2+a^2)\sin^2\theta\, d\phi^2$$

4D Kerr like (Type D!)

$$+ r^2\cos^2\theta\, d\Omega^2_{(D-4)}$$

$$\Sigma = r^2 + a^2\cos^2\theta \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

$M \propto \mu$

$J \propto Ma$ $\qquad J = \frac{2}{D-2} Ma$

Note $\quad \frac{\Delta}{r^2} = 1 - \frac{\mu}{r^{D-3}} + \frac{a^2}{r^2}$ (also valid in D=3!)

gravity attraction $\qquad$ centrifugal repulsion

The balance between The Two forces is D-dependent:
rotation refers To motion on a plane, always $\sim 1/r^2$

Horizon where $\Delta(r_H) = 0$

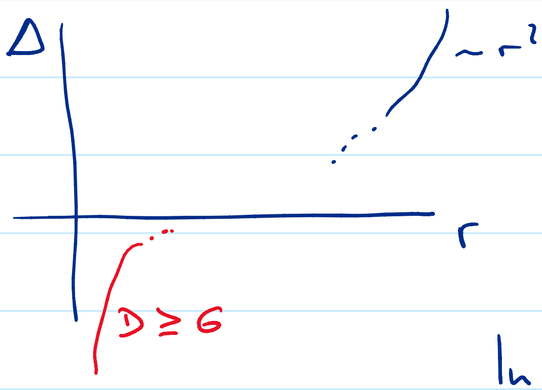
$$r_H^2 - \frac{\mu}{r_H^{D-5}} + a^2 = 0$$

only consider outer horizon
largest real root

$D=4$ $\quad r_H = \frac{\mu}{2} + \sqrt{\left(\frac{\mu}{2}\right)^2 - a^2}$ $\quad \mu/2 \geq a$ $\quad \left(\frac{\mu}{2} = M\right)$

$D=5$ $\quad r_H = \sqrt{\mu - a^2}$ $\quad \mu > a^2$ $\quad$ ($\mu = a^2$ is zero-area singularity)

$D \geq 6$



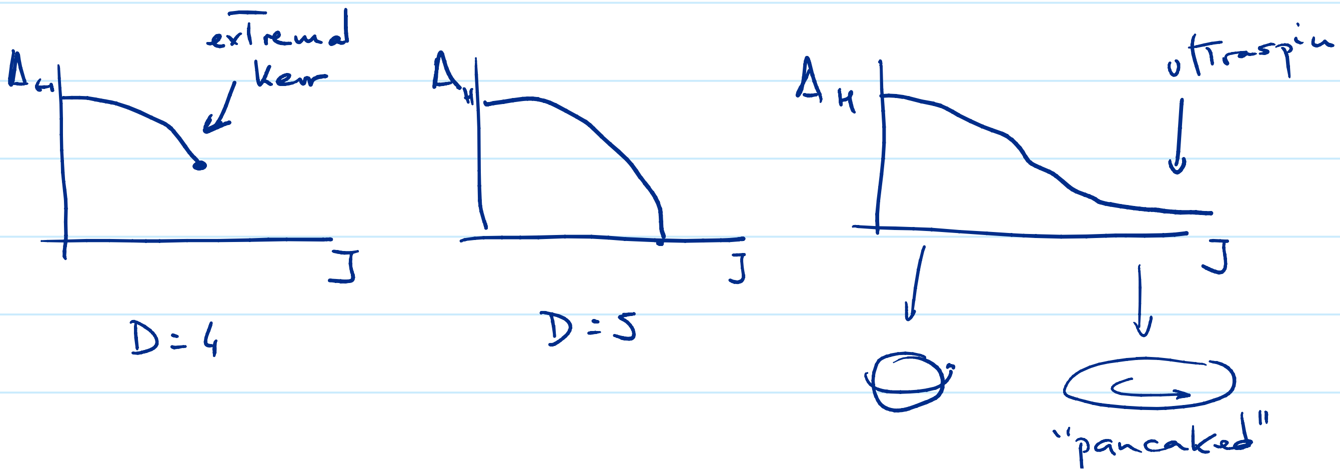
must have a root

In $D \geq 6$ there exists a horizon $\forall a$!

$\Rightarrow$ $\exists$ horizon for arbitrarily large spin, for a given mass.

Ultraspinning black holes

Fix mass:



"pancaked

The solution has a hidden symmetry (Killing-Yano Tensor) that allows to separate the r and θ variables in the massless scalar wave equation

Black strings and branes

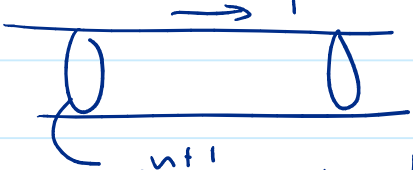
viernes, 16 de septiembre de 2022 20:24

Ricci flat × Ricci flat = Ricci flat

$R_{ij}=0$ $R_{ab}=0$ $R_{\mu\nu}=0$ $\mu=\{i\},\{a\}$

$$ds^2 = \mathrm{Schw}_{n+3} + \underbrace{d\bar{x}_p^2}_{\to \mathbb{R}^p} \qquad D=p+n+3 \qquad f=1-\frac{\mu}{r^n}$$

Black p-brane



Horizon Topology is $S^{n+1}\times\mathbb{R}^p$

Not asymptotically flat along $\bar{x}_p$

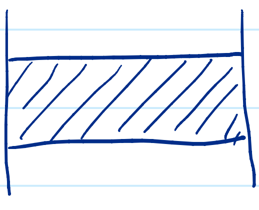
Often KK compactified : $x^i \sim x^i + L^i$

$$ds^2 = -\left(1-\frac{\mu}{r^n}\right)dt^2 + \frac{dr^2}{1-\mu/r^n} + r^2 d\Omega_{n+1} + d\bar{x}_p^2$$

Could also use MP instead of Schw

For simplicity consider $p=1$: black strings

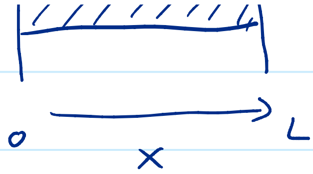
Fix L



$r_H = \mu^{1/n}$ $\qquad n=D-4$

$A_H \sim r_H^{n+1} L = \mu^{\frac{n+1}{n}} L$

$M \sim \mu L \Rightarrow \mu \sim M/L$

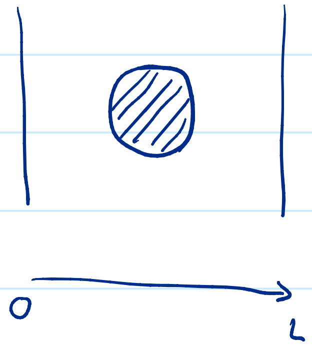


$$M \sim \mu L \Rightarrow \mu \sim M/L$$

$$A_H^{n} \sim \frac{M^{n+1}}{L}, \qquad A_H^{D-4} \sim \frac{M^{D-3}}{L}$$

$$A_H \sim M^{\frac{D-3}{D-4}} \, L^{-\frac{1}{D-4}}$$

We also expect a Tangherlini-like localized black hole

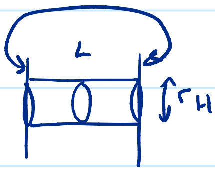


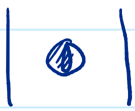
$$A_H \sim r_H^{D-2}$$

$$M \sim r_H^{D-3}$$

$$A_H \sim M^{\frac{D-2}{D-3}}$$

Set $D=5$ for simplicity





4D-like

5D

$$A_H^{\text{string}} \sim \frac{M^2}{L} \qquad A_H^{\text{loc bh}} \sim M^{3/2}$$

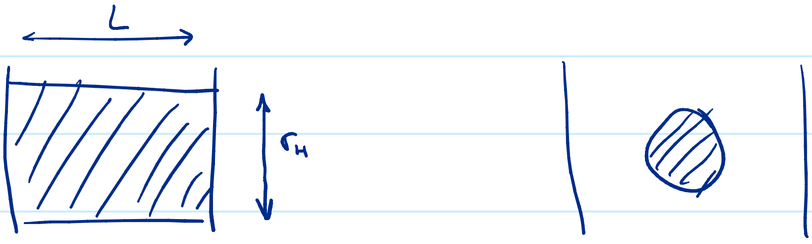
Fix M. For large enough L, $A_H^{\text{string}} < A_H^{\text{loc BH}}$

small enough L, $A_H^{\text{string}} > A_H^{\text{loc BH}}$

(must have $r_H < L$ for bh)

$$\frac{A_H^{\text{string}}}{A_H^{\text{loc bh}}} \sim \frac{M^{1/2}}{L}$$

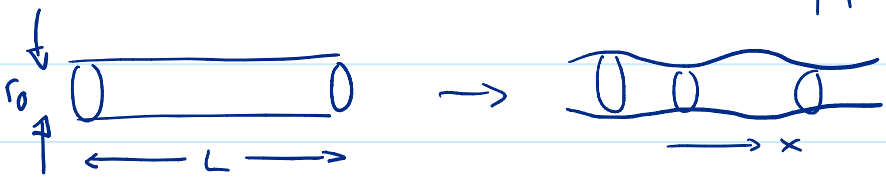
Expect dominant phase to have largest area:



$L \lesssim r_H$ $\qquad$ $L \gtrsim r_H$

$r_H < L$

Gregory + Laflamme ('93): Thin enough black strings are dynamically unstable to rippling



$r_0 \lesssim L$

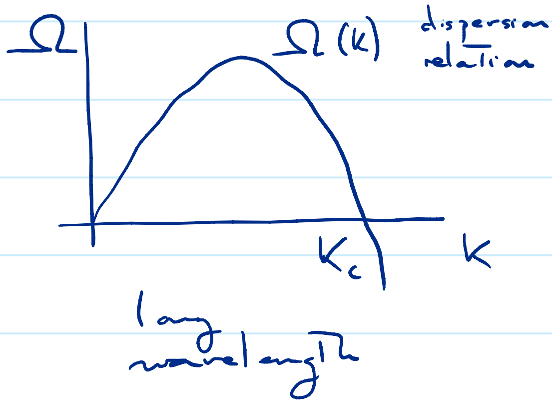
Linearized perturbation analysis (lengthy)

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

↳ black string

Find solutions of the form:

$$\delta g_{\mu\nu} = e^{\Omega t - i k x} \, h(r)$$



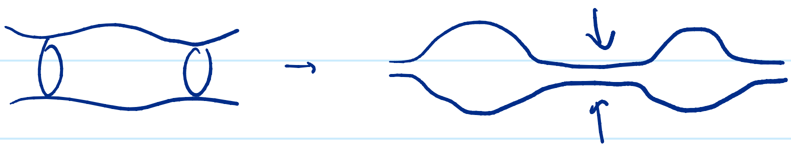
for $k < k_c \sim 1/r_H$

$\Omega > 0$: unstable

If $L > \frac{2\pi}{k_c}$ instability

This is like the Rayleigh-Plateau instability of a fluid-jet

Non-linear evolution (Lehner + Pretorius 2011)



pinch to zero size in finite time (with "satellites")

$\Rightarrow$ NAKED SINGULARITY FORMS

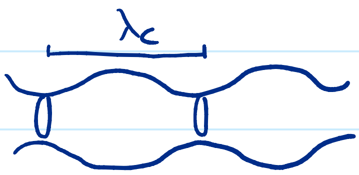
COSMIC CENSORSHIP IS VIOLATED

(but in an extremely mild manner!)

(but in an extremely mild manner!)

Quantum gravitational effects when curvature reaches Planck scale are observable from infinity

- For $k = k_c$, ie at the threshold of the instability, there is a <u>static</u>, $\Omega = 0$, perturbation that creates ripples along the horizon, w/ wavelength $\frac{2\pi}{k_c}$



non-uniform (inhomogeneous) black strings

Wiseman

Black rings and all that

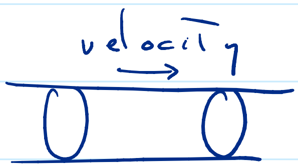
viernes, 16 de septiembre de 2022 20:24

Schwarzschild-Tangherlini and Myers-Perry are generalizations of the 4D solutions. Even if they exhibit some novel behavior, they share many properties, eg spherical horizon topology, as well as uniqueness within those families of solutions.

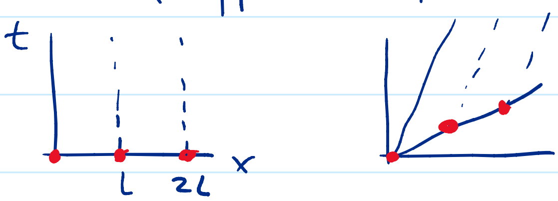
But we've learned that there exist many other solutions, some with the same topology, but also with other topologies.

Black strings can be boosted:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + dx^2 + r^2 d\Omega_{n+1} \qquad \begin{array}{l} t \to \cosh\eta\, t + \sinh\eta\, x \\ x \to \sinh\eta\, t + \cosh\eta\, x \end{array}$$



locally equivalent to static, but inequivalent if compactified (different points are identified)



boosted frame.

In more detail:

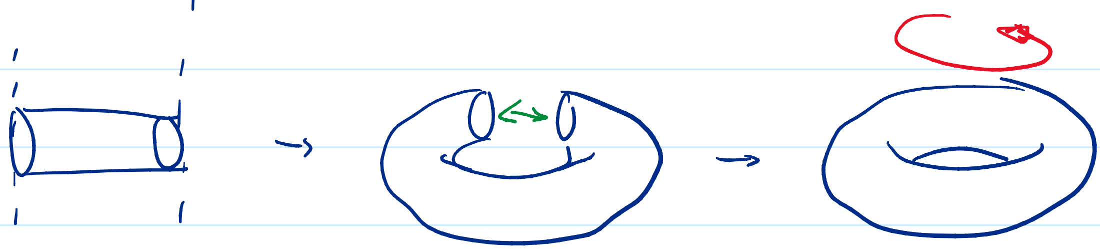
$$ds^2 = -dt^2 + dx^2 + \underbrace{\left(\frac{r_0}{r}\right)^n}_{\to\, 1-f}\left(\cosh\eta\, dt + \sinh\eta\, dx\right)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n+1}$$

more covariantly, for a black p-brane

$$ds^2 = \left(\eta_{ab} + \left(\frac{r_0}{r}\right)^n u_a u_b\right) dx^a dx^b + \frac{dr^2}{f(r)} + r^2 d\Omega_{n+1}$$

$a, b = 0, 1, \ldots, p$ w/ $u^a u^b \eta_{ab} = -1$ u^a: worldvolume velocity vector

Imagine a piece of black string: bend it to form a ring, and make it rotate, so that centrifugal repulsion balances tension and self-attraction:



This is a black ring.

Construction is possible in all $D \geq 5$

but exact solution only in $D = 5$

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - CR\frac{1+y}{F(y)}d\psi\right)^2$$

$$+\frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right]$$

$$F(\xi) = 1+\lambda\xi \qquad G(\xi) = (1-\xi^2)(1+\nu\xi)$$

$$C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}} \qquad \lambda = \frac{2\nu}{1+\nu^2}$$

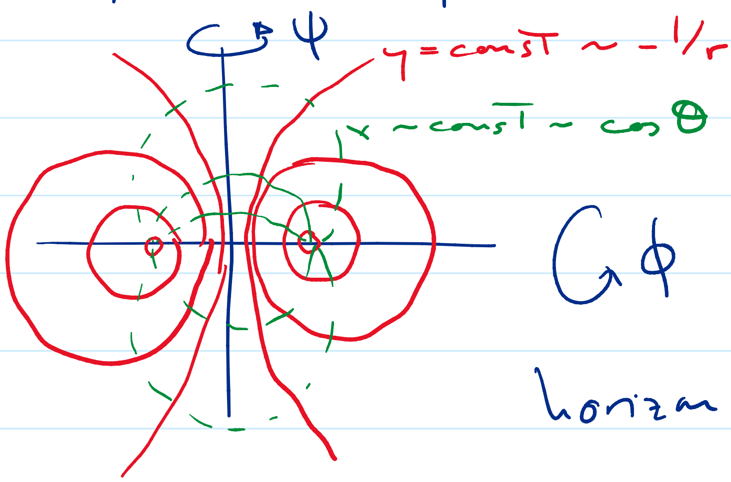
$0<\nu<1$

Two-parameter family (like MP)

R : radius parameter

$\nu \sim$ Thickness $\sim \dfrac{\text{radius}(S^2)}{\text{radius}(S^1)}$

(x, y) are adapted coordinates:



horizon at $G(y_h) = 0$

$y_h = -1/\nu$

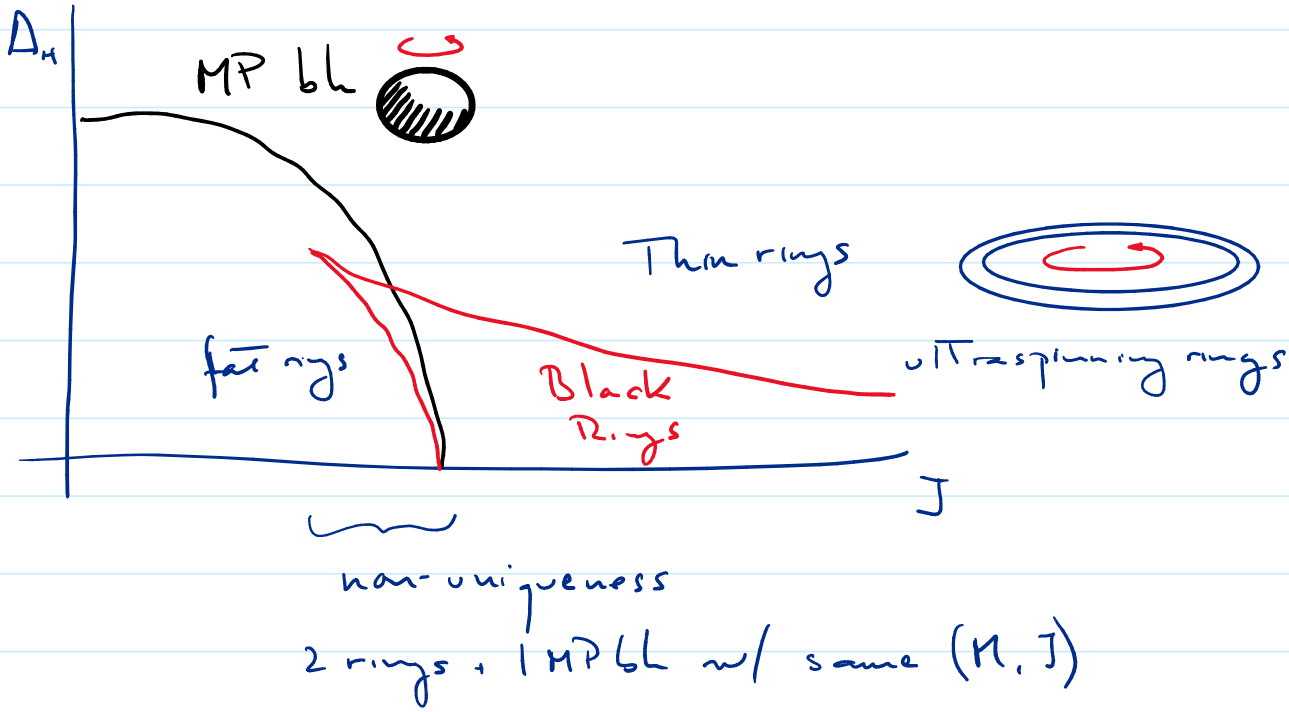
Flat space $(\nu = 0)$

$$ds^2 = -dt^2 + \frac{R^2}{(x-y)^2}\left[(y^2-1)d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right]$$

Horizon has Topology $S^1 \times S^2$

Horizon has Topology $S^1 \times S^2$

Fix mass M



The solution does not allow for separation of x and y variables in the massless scalar wave equation in general

$\exists$ black ring w/ rotation in $S^1 \times S^2$ (Pomeransky + Senkov)
(exact analytic solution)

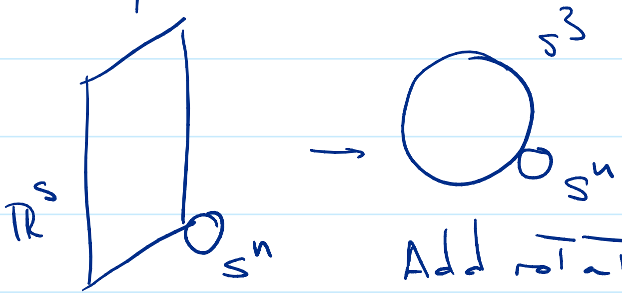
In $D>5$ only approximate solution (analytic or numerical)

In $D>5$ many other possibilities:

In $D > 5$ many other possibilities:

eg black 3-brane

$\mathbb{R}^3 \times S^n$



Add rotations in S^3 to achieve balance

(it is possible)

Instabilities

domingo, 18 de septiembre de 2022 19:12

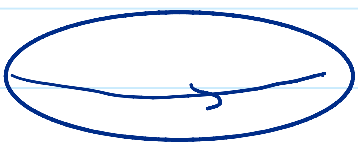
We have seen that black strings and black branes (neutral) are unstable to fluctuations with a wavelength $\lambda \gtrsim r_0$ ($k \lesssim 1/r_0$, with $2\pi = 1$)

This is the simplest instance of a more general phenomenon:

$$\boxed{\text{(neutral) horizons which are much longer along some directions than others, are unstable to rippling (and then pinching, and breaking up) along the long direction}}$$

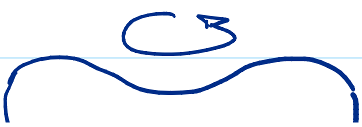
This has consequences for some higher-dimensional black holes which elongate along some directions as a consequence of spinning them up: ultraspinning instability

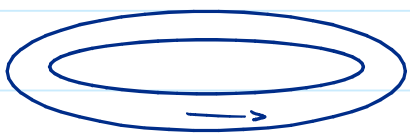
Eg



ultraspinning MP

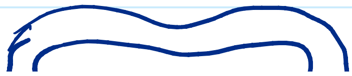
$\downarrow$

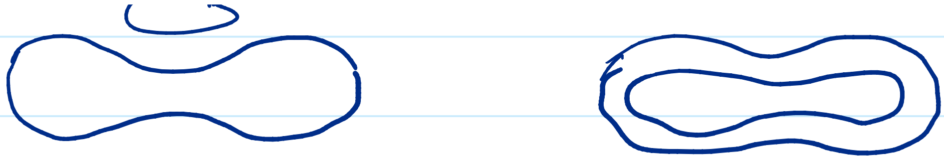




Thin black ring

$\downarrow$





This imposes a "dynamical Kerr bound" on the spin of higher-dimensional black holes — not from "formation of naked singularities" but from stability

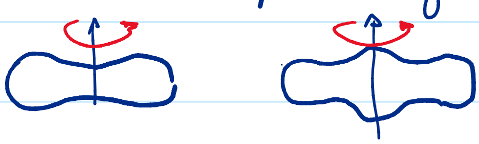
$$J \lesssim \alpha_D M^{\frac{D-2}{D-3}} \qquad (G=1)$$

α : $O(1)$ number, to be determined by explicit calculation.
Depends on D and specific black object.

All neutral black rings in 5D are dynamically unstable : when not thin : elastic instability
fat : collapse instability

More properties and consequences

- At the threshold of the instability, there's a stationary perturbation that can be continued non-linearly to give bumpy black holes:



- Other instances: "localization instability" of small $AdS_5 \times S^5$ black holes

$ds^2 = ds_5^2(\text{Sch-}AdS_5) \times S^5$

When $r_H \lesssim R_5$ the black hole of topology $S^3 \times S^5$ is unstable to rippling along the S^5, and "localizing" in S^5 to yield a 10D bh with horizon topology S^8

in AdS_5

- AdS black branes are stable: they don't have cycles to pinch-off

- Charges (including p-form charges) can improve stability, sometimes making the black holes BPS

More spins

lunes, 19 de septiembre de 2022 16:47

We saw that in $D \geq 5$ we can have rotation in more than one plane.

Amazingly, Myers+Perry managed to find exact solutions for black holes with spins in all possible planes.

I won't discuss the solutions here (they're relatively simple, but not particularly illuminating), only some of their main properties

- When all spins are turned on, $\exists$ extremal limit which has a non-singular horizon of non-zero area
- In odd D, the solution w/ all spins equal and non-zero has enhanced symmetry and only depends on the radial coordinate (it has "cohomogeneity 1").
  This is unlike Kerr, which depends on (r, θ), and simplifies a lot their study, eg perturbations, which directly reduce to ODEs

- Horizons can "pancake" (and thus become unstable)
  In even D, when at least one spin is much smaller than the other ones
  In odd D, when at least two spins are much smaller than the other ones

- The solutions have "hidden symmetries", which, like in Kerr, allow separability of variables in the scalar wave equation

Black rings in $D=5$ w/ two spins, along S^1 and S^2 have also been constructed

D>4 black holes: the takeaway

martes, 20 de septiembre de 2022 12:29

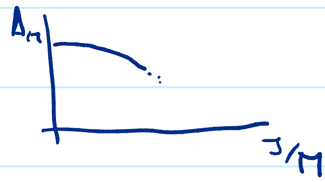
We can now organize the information we have about black holes of $R_{ij}=0$ in $D\geq 5$, according to the value of the spin for a given mass.

Define characteristic lengths for mass and spin:

$$\ell_M \sim (GM)^{\frac{1}{D-3}} \qquad \ell_J \sim J/M$$

Then we have the following regimes:

1.- $\ell_J \lesssim \ell_M$: similar to 4D Kerr
"plump" horizon
spherical topology only
dynamically stable
unique solution w/ connected horizon



2.- $\ell_J \approx \ell_M$: Threshold of new dynamics

3.- $\ell_J > \ell_M$: horizon much larger in some directions than in others
Dynamical instabilities
Non-spherical topologies
Non-unique, even w/ connected horizon

There also exist stationary multi-black hole

There also exist stationary multi-black hole solutions: black Saturns, multi-ring, etc

These bring in a lot of non-uniqueness

Black holes in D -> infinity

viernes, 16 de septiembre de 2022 20:23

As we go to higher D, more and more possibilities appear for black hole solutions (folding black branes into more complicated shapes and rotations...) so it looks like it's becoming hopelessly complicated.

However, as $D \to \infty$, even if the landscape of solutions becomes very complex, there appear dramatic simplifications in the equations that govern the dynamics of black hole horizons. They allow for efficient effective descriptions in an expansion in $1/D$, not only to construct new stationary black holes, but also to study their dynamics even non-linearly.

We won't derive the effective theory, but only discuss the origin of the simplifications that appear as $D \to \infty$

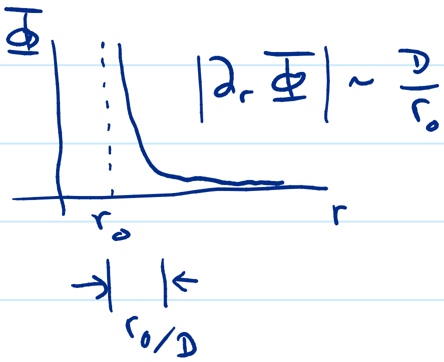
Begin w/ Schwarzschild-Tangherlini:

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\Phi = \left(\frac{r_0}{r}\right)^{D-3} \qquad \Phi \Big| \;\; |\nabla \Phi| \sim D$$

$$\Phi = \left(\frac{r_0}{r}\right)^{D-3}$$

$D \gg 1$



For $r > r_0$ $\Phi \to 0$ as $D \to \infty$

Define $R = \left(\frac{r}{r_0}\right)^n$ and keep it finite as $D \to \infty$

$n = D-3 \gg 1$

Take $r - r_0 \sim r_0/D$

Rescale $t \to \frac{2r_0}{n} t$: Time to cross near horizon region is $\sim \frac{r_0}{n}$, so rescale to finite characteristic near-horizon time

Then

$$ds^2 \to \frac{4r_0^2}{n^2}\left(-\left(1-\frac{1}{R}\right)dt^2 + \frac{1}{4R^2}\frac{dR^2}{1-1/R}\right) + r_0^2 e^{\frac{2}{n}\log R} d\Omega_n$$

This is the 2D string black hole, with dilaton

[more conventionally $R = \cosh^2\rho$

$$ds^2 = \frac{4r_0^2}{n^2}\left(-\tanh^2\rho\, dt^2 + d\rho^2\right) + r_0^2 e^{\frac{4}{n}\log\cosh\rho} d\Omega_n$$]

Indeed the spherical reduction of Einstein-Hilbert when $n \to \infty$ becomes the 2D string effective action:

for $ds^2 = g_{ij}(x)\, dx^i dx^j + r_0^2 e^{-4\Phi/n+1} d\Omega_{n+1}$

$$\underbrace{\qquad\qquad}_{2D}$$

$$I = \frac{1}{16\pi G}\int d^{n+3}x\,\sqrt{-g_{(n+3)}}\;R^{(n+3)}$$

$$= \underbrace{\frac{\Omega_{n+1}\, r_0^{n+1}}{16\pi G}}_{\text{from angle integration}}\int d^2x\,\sqrt{-g}\;e^{-2\Phi}\Big(R + \underbrace{\frac{4n}{n+1}(\nabla\phi)^2}_{\text{fluctuations of } S^{n+1}} + \underbrace{\frac{n(n+1)}{r_0^2}\,e^{4\Phi/n+1}}_{\text{curvature } S^{n+1}}\Big)$$

$$\xrightarrow{n\to\infty}\ \frac{1}{16\pi G_2}\int d^2x\,\sqrt{-g}\;e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 + 4\lambda^2\right)$$

$$G_2 = \frac{G}{\Omega_{n+1}\, r_0^{n+1}} \qquad \lambda = \frac{n}{2r_0}$$

This explains why the 2D bh looked so much like Schw and why the solution was unique: Birkhoff

This is also the universal near-horizon large-D limit of all neutral non-extremal black holes (including rotation and cosmological constant)

It underlies much of the drastic simplifications that occur in the dynamics of black holes in the limit $D\to\infty$

From Hi-D to Lo-D

viernes, 16 de septiembre de 2022 20:26

Many lo-D black holes appear in the spherical reduction of a higher D black hole, near the horizon, and often in a low-energy / low-temperature (near-extremal or not) limit.

The simplest is AdS_2 from near-horizon (near-) extremal Reissner-Nordstrom (in any D)

$$ds^2 = -\frac{f(r)}{\left(1+\frac{q}{r}\right)^2} dt^2 + \left(1+\frac{q}{r}\right)^2 \left(\frac{dr^2}{f(r)} + r^2 d\Omega_2\right)$$

$$f(r) = 1 - \frac{r_0}{r}$$

Charge $\propto q$ $\quad$ r_0 = non-extremality parameter

This is not the usual area-radius gauge:

$$\tilde{r} = r + q$$

Horizon at $r = r_0$. Area $= 4\pi (r_0 + q)^2$

Extremal limit: $r_0 = 0$, horizon at $r = 0$, finite area

In this case taking $r \ll q$, near-horizon limit:

$$ds^2 \simeq -\frac{r^2}{q^2} dt^2 + q^2 \frac{dr^2}{r^2} + q^2 d\Omega_2 \qquad r = \frac{q^2}{z}$$

$$= q^2 \left(\frac{-dt^2 + dz^2}{z^2} + d\Omega_2\right)$$

Poincaré-$AdS_2 \times S^2$

Now consider near-extremal, near-horizon

Take $r, r_0 \ll q$

$$ds^2 = -\frac{r(r-r_0)}{q^2}\,dt^2 + q^2 \frac{dr^2}{r(r-r_0)} + q^2\, d\Omega_2$$

change $r = r_0 \cosh^2 \rho/2 \qquad t = \frac{2q^2}{r_0}\tilde{t} \qquad \rho = 2\,\mathrm{arcTanh}\sqrt{\frac{r-r_0}{r}}$

$$ds^2 = q^2\left(-\sinh^2\rho\, d\tilde{t}^2 + d\rho^2 + d\Omega_2\right)$$

$\underline{\text{Rindler-}AdS_2} \times S^2$

To get Global $AdS_2 \times S^2$ we should ~~start~~ w/ a wormhole

global AdS_2 is wormhole-like

$-g_{tt} = r^2 + \kappa \qquad \kappa = -1$: gravitational attraction: +ve energy

$\kappa = +1$ " repulsion: −ve energy

One can obtain JT gravity as the Theory That describes The near-horizon dynamics of near-extremal charged black holes

(also The circle reduction of AdS_3 gravity: Achúcarro + Ortiz)

Now we obtain AdS_3 and BTZ from a charged black string in 6D (could also do in 5D)

$$ds^2 = \frac{1}{\sqrt{h_1 h_5}}\left(-dt^2 + dx^2 + \frac{r_0^2}{r^2}\left(\cosh\alpha\, dt + \sinh\alpha\, dx\right)^2\right)$$

$$+ \sqrt{h_1 h_5}\left(\frac{dr^2}{1 - r_0^2/r^2} + r^2 d\Omega_3\right) \qquad h_{1,5} = 1 + \frac{r_{1,5}^2}{r^2}$$

$$+\sqrt{h_1 h_5}\left(\frac{dr^2}{1-r_0^2/r^2}+r^2 d\Omega_3\right) \qquad h_{1,5}=1+\frac{r_{1,5}^2}{r^2}$$

x: direction along string, $x\sim x+L$

α: boost parameter along x

r_1, r_5: $\propto$ D1, D5-brane charges

r_0: non-extremality parameter

Particular cases:

$r_0\neq 0$ $\alpha=0$, $r_{1,5}=0$ $\Rightarrow$ Schwarzschild black string in $D=6$

$r_0\neq 0$ $\alpha\neq 0$, $r_{1,5}=0$ $\Rightarrow$ boosted " " " " "

$r_0\neq 0$ $r_1=r_5\neq 0$ $\alpha=0$ $\Rightarrow$ similar to RN but along a string

$r_{1,5}\neq 0$ $\underset{\alpha\to\infty}{r_0\to 0}$ $r_0\cosh\alpha\neq 0$ $\Rightarrow$ extremal D1D5P (Strominger-Vafa)

Horizon at $r=r_0$

Near-extremal: $r, r_0 \ll r_1, r_5$ $\qquad h_{1,5}\simeq \frac{r_{1,5}^2}{r^2}$

Set $\alpha=0$ for simplicity

$$ds^2\simeq \frac{r^2}{r_1 r_5}\left(-\left(1-\frac{r_0^2}{r^2}\right)dt^2+dx^2\right)+r_1 r_5\frac{dr^2}{r^2-r_0^2}+r_1 r_5\, d\Omega_3$$

rescale $(t,x)=r_1 r_5(\tau,\phi)$ $\qquad \Delta\phi=$

$$ds^2=r_1 r_5\Big[\underbrace{-(r^2-r_0^2)\,d\tau^2+\frac{dr^2}{r^2-r_0^2}+r^2 d\phi^2}_{\text{static BTZ}}+\underbrace{d\Omega_3}_{S^3}\Big]$$

static BTZ $\times$ S^3

If we have $\alpha\neq 0$ then we get rotating BTZ $\times$ S^3

If we have $\alpha \neq 0$ then we get rotating BTZ $\times S$

If $\alpha = 0$ $r_0 = 0$ we get Poincaré - $AdS_3 \times S^3$

To get Global $AdS_3 \times S^3$ we must start from a "supertube"

Observe that energy of black string above extremality $\propto r_0^2 \sim$ BTZ mass

momentum of black string $\propto$ BTZ ang mom

entropy of black string $\propto$ entropy of BTZ $\times$ area of S^3

Next we obtain the 2D string black hole from near-extremal NS5 brane

$$ds^2 = -f(r)\,dt^2 + d\vec{x}_{(5)}^2 + h_5(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3\right)$$

$$f(r) = 1 - \frac{r_0^2}{r^2} \qquad h_5 = 1 + \frac{r_5^2}{r^2} \qquad e^{2\phi} = h_5$$

↳ 10D dilaton

r_5 : $\propto$ 5-brane charge

r_0 : non-extremality parameter

$r, r_0 \ll r_5$

$$ds^2 \simeq -\left(1 - \frac{r_0^2}{r^2}\right)dt^2 + \frac{r_5^2}{r^2}\,\frac{dr^2}{1 - r_0^2/r^2} + d\vec{x}_5^2 + r_5^2\, d\Omega_3$$

$\frac{r^2}{r_0^2} = R \qquad t = r_5 \tilde{t}$

$\frac{r^2}{r_0^2} = R \qquad t = r_s \tilde{t}$

$$= r_s^2 \Big[\underbrace{-\left(1-\frac{1}{R}\right) d\tilde{t}^2 + \frac{1}{4R^2}\frac{dR^2}{1-1/R}}_{\text{2D string bh}} + \underbrace{d\Omega_3}_{\times S^3} \Big] + d\bar{x}_5^2 \qquad \times \mathbb{R}^5$$

$e^{-2\phi} = R$

This can also be seen to result from the near-extremal near-horizon limit of a 4D dilatonic magnetic black hole

Finally we obtain the black brane in AdS_5 from a near-extremal D3-brane

$$ds^2 = \frac{1}{\sqrt{h_3}}\left(-f\,dt^2 + d\bar{x}_3^2\right) + \sqrt{h_3}\left(\frac{dr^2}{f} + r^2 d\Omega_5\right)$$

$$h_3 = 1 + \left(\frac{r_3}{r}\right)^4 \qquad f = 1 - \left(\frac{r_0}{r}\right)^4$$

Again take $r_0, r \ll r$ $\qquad h_3 \to \frac{r_3^4}{r^4}$

$$ds^2 \to \frac{r^2}{r_3^2}\left(-f\,dt^2 + d\bar{x}_3^2\right) + r_3^2\left(\frac{dr^2}{r^2 f} + d\Omega_5\right)$$

$$= \underbrace{-F(r)\,dt^2 + \frac{dr^2}{F(r)} + \frac{r^2}{r_3^2} d\bar{x}_3^2}_{AdS_5 \text{ Black brane}} + \underbrace{d\Omega_5}_{S^5}$$

$$F(r) = \frac{r^2}{r_3^2} - \frac{\mu}{r^2} \qquad \mu \equiv \frac{r_0^4}{r_3^2}$$

AdS_5 black holes have $F(r) = \frac{r^2}{L^2} + k - \frac{\mu}{r^2}$

$$k = \begin{cases} 0 & \text{flat } d\vec{x}_3^2 \quad (\text{Poincaré}) \\ +1 & \text{sphere } d\Omega_3^2 \quad (\text{global}) \\ -1 & \text{hyperb } d\mathbb{H}_3 \end{cases}$$

From near-horizon limits we only get Poincaré-AdS_5.

We don't know if global-AdS_5 can be found as the near-horizon limit of an AF black brane