The asymptotic structure of gravity at spatial infinity

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where null infinity is indeed a natural place to be.

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Will give here three reasons.

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> it is important to unveil its action on spacelike (Cauchy) hypersurfaces, and thus, at spatial infinity.

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providing a standard, non-trivial, canonical realization of the BMS symmetry.

This establishes also the important fact that the BMS symmetry is a symmetry of the theory and not just a symmetry at null infinity.

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This requires understanding the action of all the BMS supertranslations at spatial infinity.

Connecting spatial infinity to the past of future null infinity (or the future of past null infinity) is actually a somewhat subtle question, because apparently "reasonable" Cauchy data might make null infinity not as smooth as one might have hoped, in the sense that the metric (and the Weyl tensor) develops logarithmic singularities.

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the realization of which exhibits (somewhat unexpectedly) a very interesting nonlinear structure.

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The analysis also provides an opportunity to develop general considerations on asymptotic symmetries.

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The definition of the theory is completed by providing boundary conditions on the dynamical variables (definition of phase space), which are assumed to make the action finite.

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Its canonical generator defines furthermore a constant of the motion.

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In gravity, all symmetries (with local action on the fields) are presumably asymptotic symmetries (no “rigid symmetry”, except perhaps non-local).

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(see R. Benguria, P. Cordero and C. Teitelboim, Nucl. Phys. B **122** (1977), 61-99)

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The moving black hole and the black hole at rest can be distinguished by the observable 4-momentum.

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Depending on the boundary conditions, these form a W_3 or an inequivalent $W_3^{(2)}$ algebra.

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Will discuss this question on the example of 4D gravity.

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The diffeomorphisms that preserve these parity conditions are the homogeneous Lorentz transformations(boosts and spatial rotations), the translations $\xi^\perp = a^0$, $\xi^i = a^i$ (a^μ = constants) and the "supertranslations" $\xi^\perp(\mathbf{n}^k)$ and $\xi^j(\mathbf{n}^k)$ of definite odd parity.

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This is indeed the case, because the standard BMS supertranslations have actually the opposite parity and thus are not allowed with the strict parity boundary conditions.

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U_{ij} and V^{ij} are the parity-twisted contributions that take the form of a gauge transformation (rewritten in Hamiltonian form).

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$$h_{ij} \equiv g_{ij} - \delta_{ij} = h^{RT}_{ij} + U_{ij}, \qquad \pi^{ij} = \pi^{ij}_{RT} + V^{ij}$$

U_{ij} and V^{ij} are the parity-twisted contributions that take the form of a gauge transformation (rewritten in Hamiltonian form).

They are of the same order as the leading terms in h_{ij} and π^{ij} ($O(1/r)$ and $O(1/r^2)$) respectively but have the opposite parity.

Parity conditions twisted by an improper diffeomorphism

To see the full BMS group, one must “unfreeze” the non-trivial BMS supertranslations,

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To preserve the boundary conditions, and for the Lorentz boosts to act canonically, one finds that U_{ij} is parametrized by an $O(1)$ odd function of the angles $\overline{U}(\mathbf{n}^k) = O(1) = -U(-\mathbf{n}^k)$ while V^{ij} is parametrized by an $O(1)$ even function of the angles $V(\mathbf{n}^k) = V(-\mathbf{n}^k)$.

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$$\xi = b_i x^i + T(\mathbf{n}) + O\left(r^{-1}\right)$$
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The zero mode of T and the first spherical harmonic component of W describe translations.

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The algebra of the generators can be easily verified to be the BMS algebra.

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while in 4 dimensions, they are at the same order but distinguished by parity conditions.

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See O. Fuentealba, M. Henneaux, J. Matulich and C. Troessaert, e-Prints : 2111.09664 [hep-th] and 2206.04972 [hep-th]

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The situation is much less understood in the infinite-dimensional case.

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The supertranslations form an abelian subalgebra, with a central extension (that vanishes for the spacetime translations).

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$$E = 2\oint_{S^3_\infty} d^3x\sqrt{\overline{\gamma}}\left[(1/2)\overline{h}^A_A + D_A\overline{\lambda}^A + 3\overline{\lambda} - (1/8)\theta_{AB}\theta^{AB} + K_A K^A\right]$$

where

$$g_{rr} = 1 + \frac{2\overline{\lambda}}{r^2} + \frac{h^{(2)}_{rr}}{r^3} + \mathcal{O}\left(r^{-4}\right), \tag{5.1}$$

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as it follows by applying standard canonical methods. (K_A comes from the momenta and non-linear redefinitions but plays no role below.)

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(first term : ADM contribution ; second term : nonlinear contribution)

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Preliminary analysis indicates that the size of the BMS group does not increase because the new terms in the diffeomorphism generators define proper gauge transformations.

Conclusions and comments

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THANK YOU !